DIRECT ADAPTIVE FUZZY CONTROL OF NONLINEAR SYSTEM CLASS WITH APPLICATIONS

H. Chekireb,* M. Tadjine,* and D. Bouchaffra**

Abstract

In this article we propose a direct adaptive fuzzy control method for MIMO nonlinear plant encountered mainly in robotics. The fuzzy adaptive law ensures the stability, convergence of the controlled outputs, and "boundedness" of adaptation parameters. In this method, the approximation error of the fuzzy logic system is estimated by an adaptive law independently of external disturbances. Moreover, the fuzzy adaptive law incorporates a compensatory sliding term, which in turn depends on this estimated error. It leads to an adaptive compensation of approximation error. Simulations for cylindrical robot and induction motor are conducted to show the effectiveness of the proposed method.

Key Words

Nonlinear plant, fuzzy adaptive law, external disturbances, dynamic error, stability analysis, Lyapunov function, three-joint robot manipulator, induction motor

1. Introduction

There is much interest in fuzzy logic systems (FLS) because of their ability to treat fuzzy variables and to induce the control law on the basis of approximate reasoning. By approximate reasoning we refer to a type of reasoning that is neither very exact nor very inexact. FLS aims at modelling the human reasoning and thinking process with linguistic variables. It are very useful when the controlled plant have some uncertainties or unknown variations. However, in order to maintain a consistent performance in the presence of real uncertainties, recourse to adaptive control is in most cases unavoidable.

Fuzzy adaptive control has been the subject of intensive research during this last decade [1–9]. Many methods have been proposed in which the fuzzy logic system is employed to approximate online the structure plant. Generally, the model obtained by fuzzy logic systems depends linearly on unknown parameters that lead to use of a Lyapunov-based learning scheme. It was first applied with success in the domain of neural networks [10, 11] because of their learning ability and universal approximating power. Hence, the combination of learning, adaptivity, and uncertainty enabled researchers to derive adaptive (or neuro) fuzzy controllers. These new models ensure the stability of the overall system and the convergence of the plant output towards a given reference signal. As an adaptive system with a finite number of parameters cannot reproduce the ideal control law, therefore an approximation error is always present. In order to compensate for this minimum approximation error and to ensure the stability in bounded state region, a sliding-mode term is incorporated in control law. The researchers in [1–3] give a solution to this inherent error problem, assuming the existence of plant upper bound. A solution, where the sliding is added to fuzzy adaptive law, is investigated in [4], whereas in [5] the adaptive FLS and SMC are used alternatively, depending on the switching conditions. The concept of persistent excitation is introduced in adaptive fuzzy control system in [6]. This concept ensures the convergence and the boundedness of adaptation parameters in the presence of approximation error and external disturbance.

The achievement H_∞ tracking performance using direct and indirect adaptive fuzzy controllers is proposed in [7]. In [8] and [9] the concept of parallel distributed compensation controller is synthesized for stable direct and indirect adaptive fuzzy control.

The upper bounds of the minimum approximation error assigned to the model and the learning coefficients are rigidly fixed by the designer. Therefore, in the presence of uncertain nonlinearities, the controller guarantees that the output tracking errors lie within a ball whose radius is proportional to the size of uncertainties. Moreover, the sliding term, used to counteract the minimum approximation error, operates by constant coefficients. Therefore, a large external disturbance may increase this error, and its elimination is possible only if the sliding term effect is sufficiently large. In some cases, the process control will probably fail, and it appears more interesting to investigate the adaptive fuzzy control law considering the unknown upper bounds on the structure plant.

In this article a new direct adaptive fuzzy control method is proposed for a class of MIMO nonlinear plant

* Laboratoire de commande des processus, Ecole Nationale Polytechnique, B.P. 182, El-Harrach, Algers, Algeria; e-mail: [chekireb, tadjine]@yahoo.fr
** Department of Computer Science and Engineering, Oakland University, Rochester MI 48309 USA; e-mail: dbouchaffra@ieee.org
(paper no. 201-1204)

encountered in robotics. This method "relaxes" the knowledge of the plant upper bounds by introducing underlying adaptation law. This law ensures the convergence and boundedness of adaptation parameters in the fuzzy systems. Furthermore, it ensures an estimation error on the plant structure by means of an adaptive algorithm independently of the external disturbances. As the compensatory sliding term itself depends on this estimated error, it leads to an adaptive compensation. By reducing the sliding term preponderance, it is possible to avoid the generation of higher-frequency mode-switching control signals that may excite high-frequency modes of unmodelled dynamics.

2. Description of the Used Fuzzy Logic System

The fuzzy logic system performs a mapping from $\boldsymbol{U}_1\times...\times\boldsymbol{U}_n \subset \boldsymbol{R}^n$ to $\boldsymbol{R}$ where each $\boldsymbol{U}_i \subset \boldsymbol{R}$, i=1,2,...,n. For the proposed fuzzy logic system we use the implication and the reasoning method suggested by Takagi and Sugeno (TS) [12]. Consequently, the fuzzy IF-THEN rules are expressed as:

$$R_k : \; if \; x_1 \; is \; A_1^{l1} \; and.......x_n \, is \; A_n^{\ln} then z_k = a^k \qquad (1)$$

where x = $(x_1,...,x_n)^T \in \boldsymbol{R}^n$ and $z_k \in \boldsymbol{R}$ are, respectively, the input of the fuzzy logic system and the consequent of the k-th rule. Here, the label A_i^{li} associated to input x_i (i= 1,2,...,n) is a fuzzy set in $\boldsymbol{U}_i$ where the indices li takes a value in $\{1, ..., m_i\}$ and m_i is the number of fuzzy sets characterizing the input x_i. The coefficient a^k (for k = 1,2,...M) is a constant coefficient of consequent part for the k-th fuzzy rule. The number of rules M is defined by the Cartesian product as: $M = m_1 \otimes m_2 \otimes .... \otimes m_n$.

In this article we employ the product operation for fuzzy implication and T-norm. The definition of the product operation is the same as in [13]. Besides, the singleton fuzzifier and weighted average deffuzzification are used. The overall output value z(x) is :

$$z(x) = \frac{\sum\limits_{k=1}^{M} \alpha_k.a^k}{\sum\limits_{k=1}^{M} \alpha_k} \qquad (2)$$

where α_k denotes the firing strength of the R_k rule, which is evaluated by using product inference and implication as:

$$\alpha_k = \prod_{i=1}^{n} \mu A_i^{li}(x_i) with l_i \in \{1, ..., m_i\} \qquad (3)$$

where $\mu A_i^{li}(x_i)$ is the membership function of x_i associated to fuzzy set A_i^{li}.

In (1), we fix the $\mu A_i^{li}(x_i)$'s and a^k'sform the adjustable parameters; z(x) can be rewritten as :

$$z(x) = w^T(x).\theta \qquad (4)$$

where θ is the parameter vector given by:

$$\theta = [a^1 \, a^2 ... \; a^M]^T \qquad (5)$$

and $w^T(x)$ is a regressive vector defined as:

$$w^T(x) = \left[\frac{\alpha_1}{\sum\limits_{k=1}^{M} \alpha_k} \; \frac{\alpha_2}{\sum\limits_{k=1}^{M} \alpha_k} \; \cdots \; \frac{\alpha_M}{\sum\limits_{k=1}^{M} \alpha_k} \right] \qquad (6)$$

In the sequel, the fuzzy logic system for multi-input single output system is represented by the mathematical model (4).

3. Problem Statement

Our goal is to build a fuzzy adaptive control system for a certain class of nonlinear dynamic systems encountered mainly in robotics. This class is of the form:

$$\begin{cases} u = F(X)x^{(p)} + G(X) \\ \qquad y = x \end{cases} \qquad (7)$$

where x = $[x_1...x_n]^T$ and the notation $x^{(p)}$ stands for the p-th order time derivative of the variable (x). Moreover, the vectors X = $[(x^{(p-1)})^T..x^T]^T$, u = $[u_1...u_n]^T$, and y = $[y_1...y_n]^T$ are, respectively, the states, the control input, and the plant outputs. The control objective is to elaborate a required control law u that forces the output y to follow their reference y_d. In order to achieve this goal, we first introduce some realistic assumptions.

Assumption A1. We assume that:

The function F(X)$\in \Re^{n\times n}$ is a positive-definite matrix fulfilling:

$$\left\| \dot{F}(X) \right\| < F_0 \left\| X \right\| \quad \forall X \in \Omega_c with F_0 > 0 \qquad (8)$$

where $\Omega_c \subseteq \Re^{n.p}$ is a subspace through which the state trajectory may travel under closed-loop control and F0 is known;

The function G(X)$\in \Re^{n\times 1}$ is a nonlinear function; it is composed of ill-known but bounded continuous functions.

The states $X \in \Re^{n.p\times 1}$are accessible.

Assumption A2: By exploiting the approximation proprieties of the fuzzy logic system defined in (4), we assume that the real nonlinear functions F(X) and G(X) are provided by the zero-order Sugeno-Tagaki fuzzy system based on M rules:

$$F(X) = W^T(X).\theta_F^\bullet + \varepsilon_F(X) \qquad (9)$$

$$G(X) = W^T(X).\theta_G^\bullet + \varepsilon_G(X) \qquad (10)$$

where $\theta_F^\bullet$, $\theta_G^\bullet$ are the best (or optimal in some sense) and $\varepsilon_F(X)$, $\varepsilon_G(X)$ are the unavoidable reconstruction errors [13].

In accordance with the employed FLS the terms W^T, θ_F, and θ_G are organized as:

$$W^T = \begin{bmatrix} w^T & 0 & 0 & 0 \\ 0 & w^T & 0 & 0 \\ ... & ... & ... & .. \\ 0 & 0 & 0 & w^T \end{bmatrix} ; \theta_F^{\bullet} = \begin{bmatrix} \theta_F^{\bullet}(1,1) & \theta_F^{\bullet}(1,2) & ... & \theta_F^{\bullet}(1,n) \\ \theta_F^{\bullet}(2,1) & \theta_F^{\bullet}(2,2) & ... & \theta_F^{\bullet}(2,n) \\ ... & ... & ... & ... \\ \theta_F^{\bullet}(n,1) & \theta_F^{\bullet}(n,2) & ... & \theta_F^{\bullet}(n,n) \end{bmatrix} \quad (11)$$

$$\theta_G^{\bullet} = \left[(\ \theta_{G1}^{\bullet})^T \ \ (\theta_{G2}^{\bullet})^T \ ... \ (\theta_{Gn}^{\bullet})^T \right]^T \quad (12)$$

where:

$$\theta_F^{\bullet}(i,j) = [a_{ij}^1 \, a_{ij}^2 ... a_{ij}^M]^T and i,j = 1..n \quad (13)$$

$$\theta_{Gi}^{\bullet} = [a_1^{Gi} \, a_2^{Gi} ... a_M^{Gi}]^T and i = 1...n \quad (14)$$

$$w^T(x) = \begin{bmatrix} \frac{\alpha_1}{\sum_{k=1}^{M} \alpha_k} & \frac{\alpha_2}{\sum_{k=1}^{M} \alpha_k} & ... & \frac{\alpha_M}{\sum_{k=1}^{M} \alpha_k} \end{bmatrix} \quad (15)$$

$$\alpha_k = \prod_{i=1}^{n} \mu A_i^{li}(x_i) for li \in \{1, ..., m_i\} \, and k \in \{1, ..., M\} \, (16) \quad (16)$$

Remark 1. Notice that, in robotics, the function F(X) is the inertia matrix, which is symmetrical and positive-definite. The bounded function G(X) represents globally the effect of Coriolis and centrifugal forces, the gravitational torques (or forces), viscous and/or dynamic friction coefficients, unstructured friction effects such as static friction terms, disturbances, or unmodelled dynamics [14].

4. The Control Synthesis

In this section, our purpose is to develop a suitable direct fuzzy adaptive control law for the plant represented by the dynamic (7). We first determine the dynamic of the filtered tracking error as a function of the fuzzy plant model, the desired trajectory, and the input vector. We next, using Lyapunov synthesis approach, search a direct fuzzy control law in order to ensure filtered tracking error convergence, the boundedness of the adaptive parameters, and all plant signals.

4.1 Tracking Dynamic Error

Let us introduce the filtered tracking error as:

$$S = [S_1 S_2 ... S_n]^T \quad (17)$$

with:

$$S_i = \left(\frac{\partial}{\partial t} + \lambda_i \right)^{(p-1)} .e_i for \lambda_i > 0 \quad (18)$$

where λ_i is positive coefficient, $e_i = x_{di} - x_i$with i=1...n, and x_{di} stands for the desired i-th output. We obtain:

$$S_i = \lambda_i^{(p-1)} .e_i + (p-1)\lambda_i^{(p-2)} \dot{e}_i + ...(p-1)\lambda_i e_i^{(p-2)} + e_i^{(p-1)}$$
$$with \ i = 1..n \quad (19)$$

Notice that $S_i = 0$ achieves the asymptotic stable tracking, as the roots of polynomial $h_i(s) = \lambda_i^{(p-1)} + (p-1)\lambda_i^{(p-2)} s + ...(p-1)\lambda_i s^{(p-2)} + s^{(p-1)}$ related to the characteristic equation of $S_i = 0$ are all in the open left-half plane via the condition $\lambda_i > 0$ with $I = 1..n$.

The relation (19) can be rewritten in the following compact form:

$$S_i = c_i^T Y_i \quad (20)$$

with:

$$Y_i = \left[e_i \ \dot{e}_i \ ... \ e_i^{(p-2)} \ e_i^{(p-1)} \right]^T \quad (21)$$

$$c_i^T = \left[\lambda_i^{(p-1)} \ (p-1)\lambda_i^{(p-2)} \ ... \ (p-1)\lambda_i \ 1 \right] \quad (22)$$

Consequently the vector S takes the form:

$$S = C^T Y \quad (23)$$

where:

$$C^T = diag \left[c_1^T \ c_2^T \ ... \ c_{n-1}^T \ c_n^T \right]_{(n \times p.n)} \quad (24)$$

$$Y = \left[Y_1^T \ Y_2^T \ ... \ Y_{(n-1)}^T \ Y_n^T \right]_{(p.n \times 1)}^T \quad (25)$$

The dynamic of S_i is given by:

$$\dot{S}_i = c_{ri}^T Y_i + e_i^{(p)} and i = 1..n \quad (26)$$

where $c_{ri}^T$is:

$$c_{ri}^T = \left[0 \ \lambda_i^{(p-1)} \ (p-1)\lambda_i^{(p-2)} \ ... \ 0.5(p-1)(p-2)\lambda_i^2 \ (p-1)\lambda_i \right] \quad (27)$$

and therefore the dynamic of S can be written into the following form :

$$\dot{S} = C_r^T Y + e^{(p)} \quad (28)$$

where:

$$C_r^T = diag \left[c_{r1}^T \ c_{r2}^T \ ... \ c_{rn}^T \ c_{rn-1}^T \right]_{(n \times p.n)} \quad (29)$$

$$e = \left[e_1 \ e_2 \ ... \ e_{n-1} \ e_n \right]^T \quad (30)$$

From relation (7) we obtain:

$$x^{(p)} = F^{-1}(X)\,(u(t) - G(X)) \tag{31}$$

$$x_d^{(p)} - x^{(p)} = x_d^{(p)} - F^{-1}(X)\,(u(t) - G(X)) \tag{32}$$

$$e^{(p)} = x_d^{(p)} - F^{-1}(X)\,(u(t) - G(X)) \tag{33}$$

Let us substitute $e^{(p)}$ given by (33) in the expression (28) of the dynamic for filtered tracking error; it follows that:

$$\dot{S} = C_r^T Y + x_d^{(p)} - F^{-1}(X)\,(u(t) - G(X)) \tag{34}$$

we define the filtered reference by:

$$Y_{ref} = x_d^{(p)} + C_r^T.Y \tag{35}$$

$$\dot{S} = Y_{ref} - F^{-1}(X)\,(u(t) - G(X)) \tag{36}$$

This is equivalent to:

$$F(X)\dot{S} = F(X)Y_{ref} + G(X) - u(t) \tag{37}$$

Using assumption A2, the filtered tracking error dynamic (37) can be transformed into the final form:

$$F(X)\dot{S} = W^T.\theta_F^{\bullet}.Y_{ref} + W^T\theta_G^{\bullet} + \varepsilon_F(t)Y_{ref} + \varepsilon_G(t) - u(t) \tag{38}$$

4.2 Stability Analysis

Proposition 1. If the system (7) is conducted by the following adaptive control law:

$$u(t) = k_d S + W^T\hat{\theta}_F Y_{ref} + W^T\hat{\theta}_G + \frac{1}{2}F_0\,\|X\|\,S + u_{sl} \tag{39}$$

where the vector parameters $\hat{\theta}_F$and $\hat{\theta}_G$ are updated by the law :

$$\dot{\hat{\theta}}_F = \gamma_2 W.S.(Y_{ref})^T \tag{40}$$

$$\dot{\hat{\theta}}_G = \gamma_1 W.S \tag{41}$$

and the sliding term u_{sl} is given by:

$$u_{sl} = (\bar{\varepsilon}_F\,\|Y_{ref}\| + \bar{\varepsilon}_G)\,sign(S) \tag{42}$$

with:

$$\bar{\varepsilon}_F \geqslant \sup_{t>0}\|\varepsilon_F(t)\|\,and\bar{\varepsilon}_G \geqslant \sup_{t>0}\|\varepsilon_G(t)\| \tag{43}$$

Then, under assumptions (A1) and (A2) we have:
S, Y, x, u, are bounded and S→0, Y→0 as t→ ∞.
$\hat{\theta}_F \to \theta_F^{\bullet}$; $\hat{\theta}_G \to \theta_G^{\bullet}$ as t→ ∞.

Proof. We consider the Lyapunov function candidate:

$$V_1 = \frac{1}{2}S^T F(X)S + \frac{1}{2\gamma_1}\tilde{\theta}_G^T\tilde{\theta}_G + \frac{1}{2\gamma_2}trace(\tilde{\theta}_F^T\tilde{\theta}_F) \tag{44}$$

with:

$$\tilde{\theta}_F = \theta_F^{\bullet} - \hat{\theta}_F and \tilde{\theta}_G = \theta_G^{\bullet} - \hat{\theta}_G \tag{45}$$

Differentiating the Lyapunov function V_1 with respect to time, we obtain:

$$\dot{V}_1 = \frac{1}{2}S^T\dot{F}(X)S + S^T F(X)\dot{S} - \frac{1}{\gamma_1}\tilde{\theta}_G^T\dot{\hat{\theta}}_G - \frac{1}{\gamma_2}trace(\tilde{\theta}_F^T\dot{\hat{\theta}}_F) \tag{46}$$

Substituting in (46) the term $F(X)\dot{S}$ by its expression (38), $\dot{V}$ becomes:

$$\begin{aligned}\dot{V}_1 = \tfrac{1}{2}S^T\dot{F}(X) + S^T\left(W^T\theta_F^{\bullet}Y_{ref} + W^T\theta_G^{\bullet} + \varepsilon_F Y_{ref} + \varepsilon_G - u(t)\right)\\ -\tfrac{1}{\gamma_1}\tilde{\theta}_G^T\dot{\hat{\theta}}_G - \tfrac{1}{\gamma_2}trace(\tilde{\theta}_F^T\dot{\hat{\theta}}_F)\end{aligned} \tag{47}$$

Now, we introduce in (47) the adaptive control law u(t) given by (39); we obtain :

$$\begin{aligned}&\dot{V}_1 = -k_d S^T S + \tfrac{1}{2}S^T\dot{F}(X)S - \tfrac{1}{2}S^T F_0\,\|X\|\,.S +\\ &S^T(\varepsilon_F Y_{ref} + \varepsilon_g) - S^T u_{sl} + (S^T W^T\tilde{\theta}_F Y_{ref})\\ &-\tfrac{1}{\gamma_2}trace(\tilde{\theta}_F^T\dot{\hat{\theta}}_F) + S^T W^T\tilde{\theta}_G - \tfrac{1}{\gamma_1}\tilde{\theta}_G^T\dot{\hat{\theta}}_G\end{aligned} \tag{48}$$

Let us introduce the parameters adaptive law (40), (41) in expression (48); this leads to:

$$\begin{aligned}&\dot{V}_1 = -k_d S^T S + \tfrac{1}{2}S^T\dot{F}(X)S - \tfrac{1}{2}S^T F_0\,\|X\|\,.S + S^T(\varepsilon_F Y_{ref} + \varepsilon_G) -\\ &S^T u_{sl} + (S^T W^T\tilde{\theta}_F Y_{ref}) - trace\left(\tilde{\theta}_F^T W S(Y_{ref})^T\right) +\\ &S^T W^T\tilde{\theta}_G - \tilde{\theta}_G^T W S\end{aligned} \tag{49}$$

As $S^T W^T\tilde{\theta}_F Y_{ref}) = trace\left((\tilde{\theta}_F)^T W S Y_{ref}\right)$, therefore $\dot{V}_1$ is reduced to:

$$\begin{aligned}&\dot{V}_1 = -k_d S^T S + \tfrac{1}{2}S^T\dot{F}(X)S - \tfrac{1}{2}S^T F_0\,\|X\|\,.S +\\ &S^T(\varepsilon_F Y_{ref} + \varepsilon_G) - S^T u_{sl}\end{aligned} \tag{50}$$

The following inequality is always fulfilled:

$$\begin{aligned}&\dot{V}_1 \leqslant -k_d S^T S + \tfrac{1}{2}S^T\left\|\dot{F}(X)\right\|S - \tfrac{1}{2}S^T F_0\,\|X\|\,.S + \|S\|\,.\\ &(\|\varepsilon_F\|\,.\,\|Y_{ref}\| + \|\varepsilon_G\|) - S^T u_{sl}\end{aligned} \tag{51}$$

By introducing the sliding terms u_{sl} given by (42), we obtain:

$$\begin{aligned}&\dot{V}_1 \leqslant -k_d S^T S + \tfrac{1}{2}S^T\left\|\dot{F}(X)\right\|.S - \tfrac{1}{2}S^T F_0\,\|X\|\,.S + \|S\|\,.\\ &(\|\varepsilon_F\|\,.\,\|Y_{ref}\| + \|\varepsilon_G\|) - \|S\|\,.\,(\bar{\varepsilon}_F.\,\|Y_{ref}\| + \bar{\varepsilon}_G)\end{aligned} \tag{52}$$

The assumption A1 and conditions (43) leads to:

$$\dot{V}_1 \leqslant -k_d S^T S < 0 \ \ \forall S \neq 0 \tag{53}$$

Therefore, the function V_1 given by (44) is a Lyapunov function for the closed-loop system (38), (40), and (41). Hence, S, $\hat{\theta}_F$,$\hat{\theta}_G$ are bounded and S→0 as t→0 [15]. As S→0, from (23), we deduce that $Y = \begin{bmatrix} e_1 \ \dot{e}_1 \ ... \ e_1^{(p-1)}, ..., \ e_n \ \dot{e}_n \ ... \ e_n^{(p-1)} \end{bmatrix}^T$ →0 as t→0. As the desired trajectories x_d and its derivatives are assumed available and bounded, we have x, X∈ L_∞.

The boundedness of control law u(t) is directly established from the boundedness of X,$\hat{\theta}_F$, and $\hat{\theta}_G$. Now, let us show that θ_F and$\theta_G \in L_2$. First let us define $V_\theta$by:

$$V_\theta = \frac{1}{\gamma_1}\int\limits_0^\infty \tilde{\theta}_G^T\tilde{\theta}_G d\tau + \frac{1}{\gamma_2}\int\limits_0^\infty trace(\tilde{\theta}_F^T\tilde{\theta}_F)d\tau \tag{54}$$

From the expression (44) of V_1, we deriveV_θ:

$$V_\theta = \int\limits_0^\infty V_1 d\tau - \frac{1}{2}\int\limits_0^\infty S^T F(X) S d\tau \tag{55}$$

As F(X) is positive-definite, the following inequality is fulfilled:

$$S^T F(X) S \geqslant \bar{\sigma}.\left\|S\right\|^2 with \bar{\sigma} = \left\|F^{-1}\right\|_\infty \tag{56}$$

Therefore, we can write:

$$V_\theta \leqslant \int\limits_0^\infty V_1 d\tau - \frac{\bar{\sigma}}{2}\int\limits_0^\infty \left\|S\right\|^2 d\tau \tag{57}$$

Note that the expressions (53), (44), and (45) mean V_1 is bounded and none is increasing with time; hence it has a finite limit:

$$\lim_{t\to\infty} V_1(S, \tilde{\theta}_F, \tilde{\theta}_G) = V_\infty < \infty and \int\limits_0^\infty V_1 d\tau \in L_\infty \tag{58}$$

Moreover, the inequality (53) leads to:

$$\int\limits_0^\infty (\left\|S\right\|^2) d\tau \leqslant -\frac{1}{k_d}\int\limits_0^\infty \dot{V}_1 d\tau \tag{59}$$

$$\int\limits_0^\infty (\left\|S\right\|^2) d\tau \leqslant \frac{1}{k_d}(V_0 - V_\infty) \in L_\infty \tag{60}$$

with $V_0 = V_1(S(0), \tilde{\theta}_F(0), \tilde{\theta}_G(0))$. The conditions (58) and (60) entail that $V_\theta \in L_\infty$, which means that $\tilde{\theta}_F$ and$\tilde{\theta}_G \in L_2$. Because W_F, W_G,Y_{ref} and $S \in L_\infty$, it follows from (40)-(41) that $\dot{\tilde{\theta}}_F$ and $\dot{\tilde{\theta}}_G \in L_\infty$ which, together with $\tilde{\theta}_F$ and $\tilde{\theta}_G \in L_2$ implies, using Barbalat lemma [15], that $\tilde{\theta}_F$ and $\tilde{\theta}_G$ →0 as t→ ∞

□

5. Case where the Bounds F $_0$, $\bar{\varepsilon}_F$, and $\bar{\varepsilon}_G$ are Ill-Known

In the precedent case, the developed adaptive control law for the nonlinear system (7) necessitates the bound F_0 of the function $\left\|\dot{F}(X)\right\|$ and bounds of the reconstruction error $\bar{\varepsilon}_F$ and $\bar{\varepsilon}_G$ associated, respectively, to the function F(x) and G(X). In this section, we assume that the bounds (F_0, $\bar{\varepsilon}_F$, $\bar{\varepsilon}_G$) are unknown, and we propose an adaptive law for these bounds such that the adaptive control law is able to "force" the plant (7) to follow the desired trajectory x_d(t).

Proposition 2. If the system (7) is conducted by the following adaptive control law:

$$u(t) = k_d S + \tfrac{1}{2}\hat{F}_0 \left\|X\right\|.S + W^T\hat{\theta}_F Y_{ref} + W^T\hat{\theta}_G + \hat{\bar{\varepsilon}}_F \left\|Y_{ref}\right\| sign(S) + \hat{\bar{\varepsilon}}_G sign(S) \tag{61}$$

where the vector parameters $\hat{\theta}_F$and $\hat{\theta}_G$ are updated by :

$$\dot{\hat{\theta}}_F = \gamma_2 W.S.(Y_{ref})^T \tag{62}$$

$$\dot{\hat{\theta}}_G = \gamma_1 W.S \tag{63}$$

and the parameter bounds $\hat{F}_0$, $\hat{\bar{\varepsilon}}_F$, $\hat{\bar{\varepsilon}}_G$ are adapted such that:

$$\dot{\hat{F}}_0 = \eta_1 \left\|X\right\|.\left\|S\right\| \tag{64}$$

$$\dot{\hat{\bar{\varepsilon}}}_F = \eta_2 \left\|Y_{ref}\right\|.\left\|S\right\| \tag{65}$$

$$\dot{\hat{\bar{\varepsilon}}}_G = \eta_2 \left\|S\right\| \tag{66}$$

where γ_1, γ_2, η_1, η_2>0.

Therefore, under assumptions (A1) and (A2) we have:

1. Y, x, u, are bounded
2. Y→0 as t→ ∞ and $\hat{\theta}_F \to \theta_F^\bullet$; $\dot{\hat{\theta}}_G \to \theta_G^\bullet$ as t→ ∞.

Proof. We consider the Lyapunov function candidate:

$$V_2 = \tfrac{1}{2}S^T F(X) S + \tfrac{1}{2\gamma_1}\tilde{\theta}_G^T\tilde{\theta}_G + \tfrac{1}{2\gamma_2}trace(\tilde{\theta}_F^T\tilde{\theta}_F) + \tfrac{1}{2\eta_1}(\tilde{F}_0)^2 + \tfrac{1}{2\eta_2}(\tilde{\bar{\varepsilon}}_F)^2 + \tfrac{1}{2\eta_2}(\tilde{\bar{\varepsilon}}_G)^2 \tag{67}$$

with:

$$\tilde{F}_0 = F_0 - \hat{F}, \quad \tilde{\bar{\varepsilon}}_F = \bar{\varepsilon}_F - \hat{\bar{\varepsilon}}_F, \quad \tilde{\bar{\varepsilon}}_G = \bar{\varepsilon}_G - \hat{\bar{\varepsilon}}_G \tag{68}$$

Differentiating the Lyapunov function V_2 with respect to time, we obtain:

$$\dot{V}_2 = \tfrac{1}{2}S^T\dot{F}(X) + S^T F(X)\dot{S} - \tfrac{1}{\gamma_1}\tilde{\theta}_G^T\dot{\hat{\theta}}_G - \tfrac{1}{\gamma_2}trace(\tilde{\theta}_F^T\dot{\hat{\theta}}_F) - \tfrac{1}{\eta_1}\tilde{F}_0\dot{\hat{F}}_0 - \tfrac{1}{\eta_2}\tilde{\bar{\varepsilon}}_F\dot{\hat{\bar{\varepsilon}}}_F - \tfrac{1}{\eta_2}\tilde{\bar{\varepsilon}}_G\dot{\hat{\bar{\varepsilon}}}_G \tag{69}$$

Substituting, in (69),$F(X)\dot{S}$ by its expression (38), $\dot{V}_2$ become:

$$\dot{V}_2 = \frac{1}{2}S^T\dot{F}(X) + S^T\left(W^T\theta_F^{\bullet}Y_{ref} + W^T\theta_G^{\bullet} + \varepsilon_F Y_{ref} + \varepsilon_G - u(t)\right)$$
$$-\frac{1}{\gamma_1}\tilde{\theta}_G^T\dot{\hat{\theta}}_G - \frac{1}{\gamma_2}trace(\tilde{\theta}_F^T\dot{\hat{\theta}}_F) - \frac{1}{\eta_1}\tilde{F}_0\dot{\hat{F}}_0 - \frac{1}{\eta_2}\tilde{\bar{\varepsilon}}_F\dot{\hat{\bar{\varepsilon}}}_F - \frac{1}{\eta_2}\tilde{\bar{\varepsilon}}_G\dot{\hat{\bar{\varepsilon}}}_G \quad (70)$$

Now, we introduce in (70) the adaptive control law u(t) given by (61); we obtain :

$$\dot{V}_2 = \frac{1}{2}S^T\dot{F}(X)S - \frac{1}{2}S^T\hat{F}_0\left\|X\right\|.S - \frac{1}{\eta_1}\tilde{F}_0\dot{\hat{F}}_0 +$$
$$S^T(\varepsilon_F Y_{ref} + \varepsilon_G) - S^T(\hat{\bar{\varepsilon}}_F\left\|Y_{ref}\right\| + \hat{\bar{\varepsilon}}_G).sign(S)$$
$$-\frac{1}{\eta_2}\tilde{\bar{\varepsilon}}_F\dot{\hat{\bar{\varepsilon}}}_F - \frac{1}{\eta_2}\tilde{\bar{\varepsilon}}_G\dot{\hat{\bar{\varepsilon}}}_G + (S^TW^T\tilde{\theta}_F Y_{ref}) - \frac{1}{\gamma_2}trace(\tilde{\theta}_F^T\dot{\hat{\theta}}_F)$$
$$+S^TW^T\tilde{\theta}_G - \frac{1}{\gamma_1}\tilde{\theta}_G^T\dot{\hat{\theta}}_G) \quad (71)$$

Introducing the parameters adaptive law (62) and (63) in expression (71) leads to:

$$\dot{V}_2 = -k_dS^TS + \frac{1}{2}S^T\dot{F}(X)S - \frac{1}{2}S^T\hat{F}_0\left\|X\right\|.S -$$
$$\frac{1}{\eta_1}\tilde{F}_0\dot{\hat{F}}_0 + S^T(\varepsilon_F Y_{ref} + \varepsilon_G) - S^T(\hat{\bar{\varepsilon}}_F\left\|Y_{ref}\right\| + \hat{\bar{\varepsilon}}_G).sign(S)$$
$$-\frac{1}{\eta_2}\tilde{\bar{\varepsilon}}_F\dot{\hat{\bar{\varepsilon}}}_F - \frac{1}{\eta_2}\tilde{\bar{\varepsilon}}_G\dot{\hat{\bar{\varepsilon}}}_G + (S^TW^T\tilde{\theta}_F Y_{rerf}) -$$
$$trace\left(\tilde{\theta}_F^T W_F S(Y_{ref})^T\right) + S^TW^T\tilde{\theta}_G - \tilde{\theta}_G^T W_G S \quad (72)$$

$\dot{V}_2$is reduced to:

$$\dot{V}_2 = -k_dS^TS + \frac{1}{2}S^T\dot{F}(X)S - \frac{1}{2}S^T\hat{F}_0\left\|X\right\|.S - \frac{1}{\eta_1}\tilde{F}_0\dot{\hat{F}}_0 +$$
$$S^T(\varepsilon_F Y_{ref} + \varepsilon_G) - S^T(\hat{\bar{\varepsilon}}_F\left\|Y_{ref}\right\| + \hat{\bar{\varepsilon}}_G).sign(S)$$
$$-\frac{1}{\eta_2}\tilde{\bar{\varepsilon}}_F\dot{\hat{\bar{\varepsilon}}}_F - \frac{1}{\eta_2}\tilde{\bar{\varepsilon}}_G\dot{\hat{\bar{\varepsilon}}}_G \quad (73)$$

The following inequality is always fulfilled:

$$\dot{V}_2 \leqslant -k_dS^TS + \frac{1}{2}S^TF_0\left\|X\right\|.S - \frac{1}{2}S^T\hat{F}_0\left\|X\right\|.S -$$
$$\frac{1}{\eta_1}\tilde{F}_0\dot{\hat{F}}_0 + \bar{\varepsilon}_F\left\|Y_{ref}\right\|.\left\|S\right\| - \hat{\bar{\varepsilon}}_F\left\|Y_{ref}\right\|.\left\|S\right\| - \frac{1}{\eta_2}\tilde{\bar{\varepsilon}}_F\dot{\hat{\bar{\varepsilon}}}_F$$
$$+\bar{\varepsilon}_G\left\|S\right\| - \hat{\bar{\varepsilon}}_G\left\|S\right\| - \frac{1}{\eta_2}\tilde{\bar{\varepsilon}}_G\dot{\hat{\bar{\varepsilon}}}_G \quad (74)$$

This is equivalent to:

$$\dot{V}_2 \leqslant -k_dS^TS + \frac{1}{2}S^T\tilde{F}_0\left\|X\right\|S - \frac{1}{\eta_1}\tilde{F}_0\dot{\hat{F}}_0 + \tilde{\bar{\varepsilon}}_F\left\|Y_{ref}\right\|.\left\|S\right\|$$
$$-\frac{1}{\eta_2}\tilde{\bar{\varepsilon}}_F\dot{\hat{\bar{\varepsilon}}}_F + \tilde{\bar{\varepsilon}}_G\left\|S\right\| - \frac{1}{\eta_2}\tilde{\bar{\varepsilon}}_G\dot{\hat{\bar{\varepsilon}}}_G \quad (75)$$

Using the adaptive law (48)–(50) of the parameter bounds$\hat{F}_0$, $\hat{\bar{\varepsilon}}_F$, $\hat{\bar{\varepsilon}}_G$, we obtain:

$$\dot{V}_2 \leqslant -k_dS^TS < 0 \ \ \forall S \neq 0 \quad (76)$$

Therefore, the function V_2 given by (67) is the Lyapunov function for the closed-loop system (38), (62)–(66). So (S, $\hat{\theta}_F$, $\hat{\theta}_G$, $\hat{F}_0$, $\hat{\bar{\varepsilon}}_F$, $\hat{\bar{\varepsilon}}_G$) are bounded and S→0 as t→0 [15]. As S→0 then from (23), $Y = \left[\begin{array}{ccccccc} e_1 & \dot{e}_1 & \ldots & e_1^{(n-1)}, \ldots, & e_n & \dot{e}_n & \ldots & e_n^{(n-1)}\end{array}\right]^T$ →0 and as x_d and its derivatives are bounded, we have x, X∈ L_∞.

The boundedness of control law u(t) is directly deduced from the boundedness of (x, X,$\hat{\theta}_f$ $\hat{\theta}_F$,$\hat{\theta}_G$, $\hat{F}_0$, $\hat{\bar{\varepsilon}}_F$, $\hat{\bar{\varepsilon}}_G$). To show that$\tilde{\theta}_F$, $\tilde{\theta}_G$,$\tilde{F}_0$, $\tilde{\bar{\varepsilon}}_f$, $\tilde{\bar{\varepsilon}}_g$ →0 as t→ ∞ we perform the same procedure (relation from (54) to (60)) by noticing that, in this case, $V_\theta$is defined by :

$$V_\theta = \frac{1}{2\gamma_1}\int_0^\infty \tilde{\theta}_G^T\tilde{\theta}_G d\tau + \frac{1}{2\gamma_2}\int_0^\infty trace(\tilde{\theta}_F^T\tilde{\theta}_F)d\tau +$$
$$\frac{1}{2\eta_1}\int_0^\infty (\tilde{F}_0)d\tau + \frac{1}{2\eta_2}\int_0^\infty (\tilde{\bar{\varepsilon}}_F)d\tau + \frac{1}{2\eta_2}\int_0^\infty (\tilde{\bar{\varepsilon}}_G)d\tau \quad (77)$$

which can be written as:

$$V_\theta = \int_0^\infty V_2 d\tau - \frac{1}{2}\int_0^\infty S^TF(X)Sd\tau \quad (78)$$

□

6. Tracking Control of a Three-Joint Robot Manipulator

In this section, the validity and effectiveness of the developed adaptive fuzzy control law are examined through the simulation of tracking control for a three-link robot manipulator. The control objective is to follow a given trajectory and to produce a torque vector such that the trajectory tracking error converges to zero. In the simulation, we examine the effects of parametric variation due to some internal uncertainties on behaviour of the closed-loop systems.

6.1 Design of Direct Adaptive Fuzzy Control System

Consider the three-joint manipulator whose dynamics is represented by:

$$\begin{cases} \tau_1 = \left[J_2 + J_3 + (m_3.(x_3 - l_3)^2 + m_0x_3^2\right].\ddot{x}_1 - \\ 2.\left[m_3.(x_3 - l_3) + m_0x_3\right].\dot{x}_1.\dot{x}_3 + f_{v1}.\dot{x}_1 \\ \tau_2 = \left[m_0 + m_2 + m_3\right].\ddot{x}_2 + \left[m_0 + m_2 + m_3\right].g + f_{v2}.\dot{x}_2 \\ \tau_3 = \left[m_0 + m_3\right].\ddot{x}_3 + \left[m_0 + m_3\right]\dot{x}_1^2 + f_{v3}.\dot{x}_3 \end{cases} \quad (79)$$

which is equivalent to dynamic plant (7) where the joint position vector x and vector X denote, respectively, $x = \left[\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}\right]^T$, $X = \left[\begin{array}{cccccc} x_1 & \dot{x}_1 & x_2 & \dot{x}_2 & x_3 & \dot{x}_3 \end{array}\right]^T$ and it appears that the function F(X) is diagonal matrix.

It easier to search the control signals when the system (79) is viewed as three subsystems, one for each link. Therefore, the dynamic plant can be rewritten:

$$u_1 = F_1(X_1)\ddot{x}_1 + G_1(X_1)withX_1 = \left[\begin{array}{ccc} \dot{x}_1 & x_3 & \dot{x}_3 \end{array}\right]^T \quad (80)$$

$$u_2 = F_2(X_2)\ddot{x}_2 + G_2(X_2)withX_2 = \dot{x}_2 \quad (81)$$

$$u_3 = F_3(X_3)\ddot{x}_3 + G_3(X_3) with X_3 = \left[\begin{array}{ccc} \dot{x}_1 & x_3 & \dot{x}_3 \end{array}\right]^T \quad (82)$$

The functions $\hat{F}_1(X_1)$, $\hat{F}_2(X_2)$,$\hat{F}_3(X_3)$, $\hat{G}_1(X_1)$, $\hat{G}_2(X_2)$, and $\hat{G}_3(X_3)$are modelled by zero-order TS fuzzy system. Each input variable is described by three fuzzy sets.

- calibration For function F_1 and G_1 , the rule base incorporates 27 rules of the form:

$$R_k^{F1}: \ if\ \dot{x}_1\ is\ A_1^{l1} and x_3\ is\ A_2^{12} and \dot{x}_3\ is\ A_3^{l3} then F_k^1 = a_k^{F1} \quad (83)$$

$$R_k^{G1}: \ if\ \dot{x}_1\ is\ A_1^{11} and x_3\ is\ A_2^{12} and \dot{x}_3\ is\ A_3^{13} then G_k^1 = a_k^{G1} \quad (84)$$

for l1, l2,l3∈ $\{1, 2, 3\}$ and k=$\{1, ..., 27\}$. The overall output is given by:

$$\hat{F}_1(x, \hat{\theta}_{F1}) = \frac{\sum_{k=1}^{27} \alpha_k^1 . F_k^1}{\sum_{k=1}^{27} \alpha_k^1} = W_{F1}^T . \hat{\theta}_{F1}\ ; \hat{G}_1(x, \hat{\theta}_{G1}) = \frac{\sum_{k=1}^{27} \alpha_k^1 . G_k^1}{\sum_{k=1}^{27} \alpha_k^1} = W_{F1}^T . \hat{\theta}_{G1} \quad (85)$$

with:

$$\alpha_k^1 = \mu A_1^{l1}(\dot{x}_1) . \mu A_2^{l2}(x_3) . \mu A_3^{l3}(\dot{x}_3) \quad (86)$$

$$\hat{\theta}_{F1} = [a_1^{F1}\ a_2^{F1}\ ...\ a_{27}^{F1}]^T \quad (87)$$

$$\hat{\theta}_{G1} = [a_1^{G1}\ a_2^{G1}\ ...\ a_{27}^{G1}]^T \quad (88)$$

- calibration For functions F_2 and G_2 the rule base is of the form:

$$R_k^{F2}: \ if\ \dot{x}_2\ is\ A_4^{l4} then F_k^2 = a_k^{F2} \quad (89)$$

$$R_k^{G2}: \ if\ \dot{x}_2\ is\ A_4^{l4} then G_k^2 = a_k^{G2} \quad (90)$$

for $l4 \in \{1, 2, 3\}$ and $k = \{1, ..., 3\}$. The overall output is given by:

$$\hat{F}_2(x, \hat{\theta}_{F2}) = \frac{\sum_{k=1}^{3} \alpha_k^2 . F_k^2}{\sum_{k=1}^{3} \alpha_k^2} = W_{F2}^T . \hat{\theta}_{F2}\ ; \hat{G}_2(x, \hat{\theta}_{G2}) = \frac{\sum_{k=1}^{3} \alpha_k^2 . G_k^2}{\sum_{k=1}^{3} \alpha_k^2} = W_{F2}^T . \hat{\theta}_{G2} \quad (91)$$

with:

$$\alpha_k^2 = \mu A_4^{l4}(\dot{x}_2) \quad (92)$$

$$\hat{\theta}_{F2} = [a_1^{F2}\ a_2^{F2}\ a_3^{F2}]^T \quad (93)$$

$$\hat{\theta}_{G2} = [a_1^{G2}\ a_2^{G2}\ a_3^{G2}]^T \quad (94)$$

- calibration For functions F_3 and G_3 the rule base is described by 27 rules of the form :

$$R_k^{F3}: \ if\ \dot{x}_1\ is\ A_1^{l1} and x_3\ is\ A_2^{l2} and \dot{x}_3\ is\ A_3^{l3} then F_k^3 = a_k^{F3} \quad (95)$$

$$R_k^{G3}: \ if\ \dot{x}_1\ is\ A_1^{l1} and x_3\ is\ A_2^{l2} and \dot{x}_3\ is\ A_3^{l3} then G_k^3 = a_k^{G3} \quad (96)$$

for l1, l2,l3∈ $\{1, 2, 3\}$ and k = $\{1, ..., 27\}$.
Therefore, the overall output is given by:

$$\hat{F}_3(x, \hat{\theta}_{F3}) = \frac{\sum_{k=1}^{27} \alpha_k^3 . F_k^3}{\sum_{k=1}^{27} \alpha_k^3} = W_{F3}^T . \hat{\theta}_{F3}\ ; \hat{G}_3(x, \hat{\theta}_{G3}) = \frac{\sum_{k=1}^{27} \alpha_k^3 . G_k^3}{\sum_{k=1}^{27} \alpha_k^3} = W_{F3}^T . \hat{\theta}_{G3} \quad (97)$$

with:

$$\alpha_k^1 = \mu A_1^{l1}(\dot{x}_1) . \mu A_2^{l2}(x_3) . \mu A_3^{l3}(\dot{x}_3) \quad (98)$$

$$\hat{\theta}_{F3} = [a_1^{F3}\ a_2^{F3}\ ...\ a_{27}^{F3}]^T \quad (99)$$

$$\hat{\theta}_{G3} = [a_1^{G3}\ a_2^{G3}\ ...\ a_{27}^{G3}]^T \quad (100)$$

Consequently, the control inputs u_j (for j=1,. . . ,3) are determined by:

$$u_j(t) = k_{dj} S_j + \tfrac{1}{2} \hat{F}_{0j} \left\| X_j . \right\| . S_j + W_{Fj}^T \hat{\theta}_{Fj} Y_{jref} + W_{Gj}^T \hat{\theta}_{Gj} + u_{sj} \quad (101)$$

with:

$$S_j = (\dot{x}_{jref} - \dot{x}_j) + \lambda_j (x_{jref} - x_j) \quad (102)$$

$$Y_{jref} = \ddot{x}_{jref} + \lambda_j (\dot{x}_{jref} - \dot{x}_j) \quad (103)$$

$$u_{sj} = \left(\hat{\bar{\varepsilon}}_{Fj} \left| Y_{jref} \right| + \hat{\bar{\varepsilon}}_{Gj} \right) sign(S_j) \quad (104)$$

where the parameters are updated by the following adaptive law:

$$\dot{\hat{\theta}}_{Fj} = \gamma_j W_{Fj}.S_j.(Y_{jref})^T \quad (105)$$

$$\dot{\hat{\theta}}_{Gj} = \gamma_j W_{Gj}.S_j \quad (106)$$

and the unknown bounds are corrected by :

$$\dot{\hat{F}}_{0j} = \eta_j \left|X_j\right| \left|S_j\right| \quad (107)$$

$$\dot{\hat{\varepsilon}}_{Fj} = \eta_j \left|Y_{jref}\right| \left|S_j\right| \quad (108)$$

$$\dot{\hat{\varepsilon}}_{Gj} = \eta_j \left|S_j\right| \quad (109)$$

6.2 Simulation Results

The control law is constituted by an adaptive fuzzy model term ($W_{Fj}^T \hat{\theta}_{Fj} Y_{jref} + W_{Gj}^T \hat{\theta}_{Gj}$), the metric term ($k_{dj} S_j + \frac{1}{2} \hat{F}_{0j} \left\|X_j\right\| S_j$), and the sliding compensatory term (u_{sj}). It is imperative to look for the control coefficients such that the adaptive fuzzy model term is preponderant. Satisfactory results are obtained for the control coefficients and bounds initial value ($\hat{F}_{0j}$, $\hat{\varepsilon}_{Fj}$, $\hat{\varepsilon}_{Gj}$), which are set up as indicated in Table 1.

Table 1
Control Coefficients and Initial Bounds

Joint	k_d	λ	γ	η	$\hat{F}_0$	$\hat{\varepsilon}_F$	$\hat{\varepsilon}_G$
n ° 1	1500	100	100	1.2	1	0.2	0.2
n ° 2	25000	100	1	1.2	1	1	1
n ° 3	750	75	2000	10	1	0.5	0.5

The simulation is conducted when the desired trajectory for each joint is a cycloid given by:

$$q_{refj} = q_{0j} + \frac{(q_{fj} - q_{0j})}{2\pi} \left(\frac{2\pi t}{t_f} - \sin(\frac{2\pi t}{t_f}) \right) for j = (1, 2, 3) \quad (110)$$

where q_{0j}, q_{fj}, and t_f are, respectively, initial position joint value, final position joint value, and final time.

The simulations are conducted, first, where the robot works in empty regime (test1: see Fig. 1); second, where the robot is initially loaded with a 10kg mass that is let go at time $t = 1$s (test2: see Fig. 2); and third, where the robot starts in empty regime after which all parameters are increased with 50% around nominal values at the time $t =$ 1s (test3: see Fig. 3). The obtained results show that the tracking regime is effectively established with acceptable tracking errors (less than $2{\times}10^{-3}$) and the control inputs (u_1, u_2, u_3) appear feasible. When the robot is in test1 the tracking errors and control inputs take their smallest values. Moreover, the inputs remain continuous (see Fig. 1). However, the control inputs increase somewhat relative to test1 and present an acceptable discontinuity at time $t =$ 1s when the 10kg mass is slipping and when the parametric variations are applied.

Figure 1. The robot responses to test1.

Figure 2. The robot responses to test2.

Figure 3. The robot response to test3.

Table 2 shows the maximum control inputs in the three regimes. It appears that for the first joint maximal input $(u_1)_{max}$ remains practically constant and $(u_2)_{max}$ increases by 31.5% in test2 to 41% in test3. For the third joint, $(u_3)_{max}$ increases by 29.5% in test 2 to 40.7% in test3. The absolute maximal tracking errors concerning the first and the second joint remain practically unchanged for these three tests. Meanwhile, the third joint maximum tracking error increases by 10% in test2 to 20% in test3 (see Table 2). The adaptive sliding terms (u_{s1}, u_{s2}) keep sensitively the same form and the same absolute maximum value; meanwhile, u_{s3} shows more shattering in test3.

Theses results reveals that the direct adaptive fuzzy control law is highly robust in the face of internal uncertainties and external disturbances. Moreover, because the sliding terms evolve in practically the same interval in the three cases, consequently the adaptive fuzzy model term remains the preponderant term in the control law.

Table 3
The Maximal Absolute Value of Sliding Terms

	$\left\|u_{s1}\right\|_{\max}$	$\left\|u_{s2}\right\|_{\max}$	$\left\|u_{s3}\right\|_{\max}$
Test1	2.08	12.90	6.63
Test2	2.09	12.91	7.84
Test3	2.08	12.92	7.35

7. Application to Induction Motor

7.1 Control Law

In (d,q) reference frame fixed to rotor field, the dynamic of the flux ϕ_r and the rotating speed pulsation (ω_r) of the three-phase induction motor are reduced to the following equations [16], where (I_d I_q) are the components of the stator current:

$$\begin{cases} I_d = \frac{T_r}{M}\frac{d\phi_r}{dt} + \frac{1}{M}\phi_r \\ I_q = \frac{JL_r}{p^2 M \phi_r}\frac{d\omega_r}{dt} + \frac{L_r k_f}{p^2 M \phi_r}\omega_r + \frac{L_r p}{p^2 M \phi_r}\Gamma_r \end{cases} \quad (111)$$

where the motor parameters represent:

. T_r : rotor electric constant

Table 2
Maximal Absolute Tracking Errors and Maximal Inputs

	$(\|er_1\|)_{max}$	$(\|er_2\|)_{max}$	$(\|er_3\|)_{max}$	$(u_1)_{max}$	$(u_2)_{max}$	$(u_3)_{max}$
Test1	1.5×10^{-3}	0.9×10^{-3}	10^{-3}	18.35	358.76	36.97
Test2	1.5×10^{-3}	0.9×10^{-3}	1.1×10^{-3}	20.17	472	47.88
Test3	1.6×10^{-3}	0.9×10^{-3}	1.2×10^{-3}	18.35	505.29	52.01

. L_s, L_r: stator cyclic inductance, rotor cyclic inductance
. M: mutual inductance cyclic between stator and rotor
. k_f : viscous coefficient
. J: inertia
. Γ_r : load torque
. p : pair of poles

In the field rotating reference frame, it is well known that the dynamic of flux depends only on the control input I_d and the flux ϕ_r. Moreover, the dynamic of the speed depends mainly on the control input I_q and the speed ω_r. The influence of the flux on the dynamic of the speed is not taken into account. Therefore, the investigation of the control inputs is based on the following simple system:

$$\begin{cases} I_d = F_1(\phi_r)\dot{\phi}_r + G_1(\phi_r) \\ I_q = F_2(\omega_r)\dot{\omega}_r + G_2(\omega_r) \end{cases} \tag{112}$$

For the first system, the estimation of the functions F_1 and G_1 is carried out with only three rules, so the flux is described by three fuzzy sets. By contrast, for the second system, the reconstruction of the functions F_2 and G_2 is obtained with five rules, and hence the speed is described by five fuzzy sets.

Using the latter procedure, we compute the control input as follows:

$$\begin{aligned} &I_d = k_{d1}S_1 + 0.5.F0_1\,|\phi_r|\,S_1 + (W_{F1})^T\hat{\theta}_{F1}Y_{ref1} + \\ &(W_{G1})^T\hat{\theta}_{G1} + \hat{\bar{\varepsilon}}_{F1}\,|Y_{ref1}|\,sign(S_1) + \hat{\bar{\varepsilon}}_{G1}sign(S_1) \\ &I_q = k_{d2}S_2 + 0.5.F0_2\,|\omega_r|\,S_2 + (W_{F2})^T\hat{\theta}_{F2}Y_{ref2} + \\ &(W_{G2})^T\hat{\theta}_{G2} + \hat{\bar{\varepsilon}}_{F2}\,|Y_{ref2}|\,sign(S_2) + \hat{\bar{\varepsilon}}_{G2}sign(S_2) \end{aligned} \tag{113}$$

where:

$$S_1 = \phi_{ref} - \phi_r; \quad S_2 = \omega_{ref} - \omega_r \tag{114}$$

$$Y_{ref1} = \dot{\phi}_{ref} + \lambda_1(\phi_{ref} - \phi_r); \quad Y_{ref2} = \dot{\omega}_{ref} + \lambda_2(\omega_{ref} - \omega_r) \tag{115}$$

The parameters are updated by the laws:

$$\dot{\hat{\theta}}_{G1} = \gamma_{11}W_{G1}S_1 \quad \dot{\hat{\theta}}_{G2} = \gamma_{12}W_{G2}S_2 \tag{116}$$

$$\dot{\hat{\theta}}_{F1} = \gamma_{21}W_{F1}S_1Y_{ref1} \quad \dot{\hat{\theta}}_{F2} = \gamma_{22}W_{F2}S_2Y_{ref2} \tag{117}$$

whereas the upper bounds are estimated by the laws:

$$\dot{\hat{F}}0_1 = \eta_1\,|\phi_r|\,|S_1|\,; \quad \dot{\hat{F}}0_2 = \eta_2\,|\omega_r|\,|S_2| \tag{118}$$

$$\dot{\hat{\bar{\varepsilon}}}_{F1} = \eta_1\,\|Y_{ref1}\|\,.\,|S_1|\,; \quad \dot{\hat{\bar{\varepsilon}}}_{F2} = \eta_2\,\|Y_{ref2}\|\,.\,|S_2| \tag{119}$$

$$\dot{\hat{\bar{\varepsilon}}}_{G1} = \eta_1\,|S_1|\,; \quad \dot{\hat{\bar{\varepsilon}}}_{G2} = \eta_2\,|S_2| \tag{120}$$

The controls $u(t) = (I_d, I_q)$ are the (d,q) components of the stator current. Because the three-phase induction motor is voltage fed, it is necessary to determine the required stator voltage components (u_α, u_β) that effectively impose this stator current. This is achieved by feeding the motor with three-phase current controlled inverter. The structure of this control scheme is shown in Fig. 4. In this figure, the adaptive fuzzy controller (AFC) for flux elaborates the control law I_d from variables $\left(\dot{\phi}_{ref}\ \phi_{ref}\ \phi_r\right)$ and the adaptive terms $\hat{F}_1$, $\hat{G}_1$,$\hat{F}0_1$, $\hat{\bar{\varepsilon}}_{F1}$ and $\hat{\bar{\varepsilon}}_{G1}$. With the same procedure, the adaptive fuzzy controller for speed computes the control signal I_q from variables $\left(\dot{\omega}_{ref}\ \omega_{ref}\ \omega_r\right)$ and the adaptive terms $\hat{F}_2$, $\hat{G}_2$,$\hat{F}0_2$, $\hat{\bar{\varepsilon}}_{F_2}$, and $\hat{\bar{\varepsilon}}_{G_2}$. In order to maintain the stator current in acceptable range, the control inputs (I_d, I_q) are transformed into limited inputs (I_d^*, I_q^*). The three-phase reference current ($i_a^\bullet$, $i_b^\bullet$, $i_c^\bullet$) is obtained from the (d, q) stator reference current (I_d^*, I_q^*) by using the inverse Park transformation (Tp^{-1}). The actual stator current (i_a, i_b, i_c) is restricted in hysteresis bandwidth Δi around the three-phase references currents by using an appropriate switching of the inverter legs. The induction motor is modelled by its five nonlinear differential equations in stator reference frame (α, β), including the stator current components (i_α, i_β), the rotor field components (ϕ_α, ϕ_β), and the speed rotor ω_r[16]. The rotor field position (Θ), which is used in the Park transformation (Tp) and its inverse (Tp^{-1}), is assumed available from measurement (or observation).

Figure 4. Direct adaptive fuzzy control scheme of induction motor.

7.2 7.2 Simulation Results

The motor under tests is characterized by:
P = 3.7Kw, 220/380V, 8.54/14.8A
M = 0.048H, L_s= 0.17H, L_r= 0.015H, $\sigma = 0.0964$
T_s= 0.151s, T_r= 0.136s, J = 0.135mN/rds^{-2}, K_f= 0.0018mN/rdS^{-1}

The current-controlled inverter is fed by 600V continue voltage assumed constant, and the hysteresis bandwidth of stator current controller is fixed to 1A. The reference signals of flux and speed vary in physical size in the range [0, ϕ_n] and [-1.33ω_n,1.33ω_n],respectively, where ϕ_n is nominal flux and ω_n the nominal speed. In order to have some admissible errors, these intervals should be extended, such as [-0.1ϕ_n, 1.1ϕ_n] for flux and [-1.5ω_n,1.5ω_n] for speed. It is obvious that these intervals become, in relative value, [-0.1, 1.1] for flux and [-1.5,1.5] for speed. Therefore, the fuzzy sets of flux and of speed are regularly distributed on the universe of discourse [–0.1, 1.1] and [–1.5, 1.5], respectively. The desired flux and speed tracking are involved with the regulator coefficients tuned to values given in Table 4:

Table 3
Control Coefficients and Initial Bounds

	k_d	λ	γ	η	$\hat{F}_0$	$\hat{\bar{\varepsilon}}_F$	$\hat{\bar{\varepsilon}}_G$
Flux	5	5	1	1.2	1	1.2	1.2
Speed	5	5	1	1.2	1	1.2	1.2

The flux and speed-tracking responses of induction motor, under the proposed fuzzy adaptive control, are shown in Fig. 5 (for both ω_{ref}>0 and ω_{ref}<0). These responses are obtained in the case where the nominal load torque and parametric variations are applied (at the same moment) during 1s respectively at the time t = 0.85, 1.3s, and 2.05s. The parameter variations are involved around nominal values as that the stator and rotor resistors increase, respectively, by an amount of 50% and 100%, all stator and rotor inductors decrease respectively by an amount of 30%. Moreover, in field-weakening regime (i.e., when the speed reference ω_{ref} grows up to nominal value ω_n= 300rd/s), the reference flux ϕ_{ref} is reduced down to the nominal value ϕ_n (ϕ_n = 0.33Wb) as:$\phi_{ref} = \phi_n\omega_n/\omega_{ref}$.

It appears that the flux and speed track their references with a good accuracy and good precision despite the presence of strong disturbances, as the maximal tracking errors reported to their nominal value remain weak 1.5% for flux and 0.14% for speed (see Table 5). Meanwhile, the tracking regime is lost in the beginning of the transient stage (for t<0.12s) due to the fact that the control inputs (I_d I_q) are in limitation regime.

Table 4
Maximum Tracking Errors

Max. Tracking Error	In Case ω_{ref}>0	In Case ω_{ref} <0
$\lvert\phi_{ref} - \phi\rvert$	4.6×10^{-3}Wb	5×10^{-3}Wb
$\lvert\omega_{ref} - \omega\rvert$	0.44rd/s	0.33 rd/s

Figure 5. Flux-speed tracking of induction motor.

8. Conclusion

In this article we have designed a direct adaptive fuzzy control law for a class of nonlinear system encountered in robotics. This control law ensures the convergence of tracking errors and boundedness of the fuzzy logic system parameters and all signal plants. This law incorporates an adaptive sliding term to compensate the unknown minimum approximation error between the fuzzy logic model and the controlled plant. This compensation is performed independently of internal or external disturbances. The application of the developed method is carried out for a cylindrical joint robot and induction motor. The obtained simulation results shows that this direct adaptive fuzzy control law maintains the tracking errors in an acceptable interval with feasible control inputs in the presence of hard parameter variations or external disturbances.

References

[1] L.X. Wang, Stable adaptive fuzzy control of nonlinear systems, *IEEE Trans. on Fuzzy Systems, 1*(1), 1993, 146–155.

[2] L.X. Wang, *Adaptive fuzzy systems and control: Design and stability* (Englewood Cliffs, NJ: Prentice-Hall, 1994).

[3] C.Y. Su & Y. Stephanenko, Adaptive control of a class of non linear system with fuzzy logic, *IEEE Trans. Fuzzy System, 2*(2), 1994, 285–294.

[4] Y.S. Lu & J.S. Chen, A self-organizing fuzzy sliding-mode controller design for a class of nonlinear servosystems, *IEEE Trans. on Industrial Electronics, 41*(3), 1994, 492–502.

[5] H.C. Myung & Z.Z. Bien, Stabilization of direct adaptive fuzzy control systems considering minimum approximation error, *Proc. IEEE Int. Conf. on Fuzzy Systems*, Korea, 1999, 257–262.

[6] Y.T. Kim & Z. Bien, Robust indirect adaptive fuzzy control, *Proc. IEEE Int. Conf. on Fuzzy Systems*, Korea, 1999, 1293–1298.

[7] B.S. Chen, C.H. Lee, & Y.C. Chang, $H^{\propto}$ tracking design of uncertain nonlinear SISO systems: Adaptive fuzzy approach, *IEEE Trans. Fuzzy Systems, 4*(1), 1996, 32–43.

[8] Y.W. Cho, Y.H. Yee, & M. Park, An indirect model reference adaptive fuzzy control for SISO Takagi-Sugeno model, *Proc. IEEE Int. Conf. on Fuzzy Systems*, Korea, 1999, 474, 479.

[9] Y.W. Cho, E.S. Kim, K.C. Lee, & M. Park, Tracking control of a robot manipulator using a direct model reference adaptive fuzzy control, *Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, 1999, 100–105.

[10] E. Tzirkel-Hancock & F. Fallside, A direct control method for a class of nonlinear systems using neural networks, *Proc. 2nd Inst. Elect. Eng. Int. Conf. Artificial Neural Networks*, Bournemouth, U.K., 1991, 134-138.

[11] R.M. Sanner & J.-J.E. Slotine, Gaussian networks for direct adaptive control, *IEEE Trans. Neural Networks, 3*, 1992, 837–863.

[12] T. Takagi & M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, *IEEE Trans. Systems, Man and Cybernetics, 15*(1), 1985, 116–132.

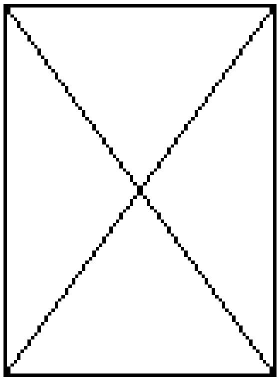
[13] L.X. Wang, Design and analysis of fuzzy identifiers of nonlinear dynamics systems, *IEEE Trans. on Automatic Control, 40*(1), 1995.

[14] J.J. Craig, *Introduction to robotics: Mechanics and control*, 2nd ed. (Addison-Wesley) 1986.

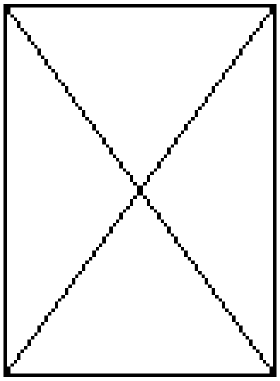
[15] S. Sastry & M. Bodson, *Adaptive control: Stability convergence, and robustness* (Englewood Cliffs, NJ: Prentice-Hall, 1989).

[16] W. Leonhard, *Control of electric drive* (New York: Springer Verlag, 1990).

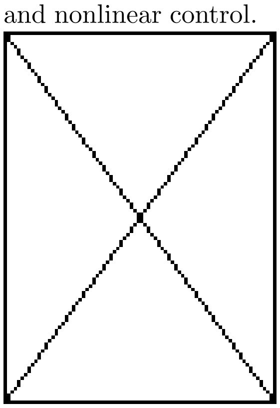
Biography



Hachemi Chekireb received his engineering degree and M.Sc. in power electronics from the National Polytechnic School of Algiers (ENP), Algeria, in 1979 and 1986, respectively. Since 1987 he has held teaching and research positions in the Department of Electrical Engineering, ENP, where he is currently Assistant Professor. His research interests are in electrical drives and fuzzy systems.



Mohamed Tadjine received his engineering degree from the National Polytechnic School of Algiers (ENP), Algeria, in 1990 and the Ph.D. degree in automatic control from the National Polytechnic Institute of Grenoble (INPG), France, in 1994. From 1995 to 1997, he was a researcher at the Automatic Systems Laboratory, Amiens, France. Since 1997 he has been with the Department of Automatic Control, ENP, where he is currently Associate Professor. His research interests are in robust and nonlinear control.



Djamel Bouchaffra obtained his Ph.D. in computer science from Grenoble University, France, in 1992. He is currently Assistant Professor of Computer Science and Engineering at Oakland University, USA, teaching pattern recognition, artificial intelligence, soft computing, and discrete mathematics. His field of research is within these areas. He has written several papers for peer-reviewed conferences and premier journals and chaired a number of sessions in prestigious conferences. He is a regular reviewer for many journals, such as IEEE Trans. PAMI, TKDE, and Image Processing. Professor Bouchaffra is a senior member of the IEEE, a member of the Computer Society, and the vice chair of the IEEE/SEM chapter V.