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A relation between isometries and the relative consistency concept in probabilistic logic

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Abstract

The paper experiments some topological features in analyzing Nilsson's probabilistic logic. This approach provides an extension of Nilsson's approach: by exploring different norms in the space of possible worlds and the space of sentences. The case of the infinite norm, the L_1 norm and the Euclidean norm is studied and analyzed. However, by using the isometries concept, we found out that the space of sentences is a probability space. We also demonstrate that the classical representation (0-1) of the truth values associated to predicates is not suitable for determining the consistent region.

1. INTRODUCTION

A lot of mathematical work [1] on probabilistic inference already exists. For instance, the expert system Mycin handled uncertain knowledge, the Prospector system used a method based on Bayes' rule. The probabilistic logic introduced by Nilsson encounters some difficulties. We focus only on the problem of the consistent region associated to a set of predicates. The questions that one can ask are:
(i) when is the space of predicates a probability space?
(ii) knowing the probabilities associated to (n - 1) predicates, what is the probability associated to the nth predicate?
(iii) if the latter probability is not unique, what do we do?

In this paper, in order to answer these questions, one first provides an extension of Nilsson's probabilistic logic [2]: by exploring different metrics. We secondly show the role played by the isometries concept. The second section defines the notion of consistency and the probability concept. We expose in the third section the algebraic and geometric interpretation. From the fourth until the eighth section, we present several ways of obtaining the consistent region according to metrics. We explore the infinite norm, the L_1 norm, the Euclidean norm and also their corresponding isometries.

2. THE CONSISTENCY AND PROBABILITY CONCEPT

We define these concepts by an example : let's consider the following set of sentences S={P,¬PvQ,Q}, so we expect to obtain among all possible worlds (2^3) just 2^2 consistent possible worlds; other possible worlds are inconsistent. We can write the matrix corresponding to the semantic tree, we call this matrix "the consistent matrix C",

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

We define the probability of S as, $P(S) = \Sigma P(V_i)*\psi_{Vi}(S)$ where $P(V_i)$ is the probability that the actual world V_a is equal to V_i, the characteristic function ψ is defined as: $\psi_{Vi}(S) = 1$ if S is true in V_i ; 0 otherwise. The V_i's are the column vectors of the matrix C, i belongs to the set {1,2,3,4}, S is any sentence. This definition of the probability associated to a sentence gives a mathematical and geometric interpretation of the consistent area in the space of the 3 sentences.

3. THE ALGEBRAIC AND GEOMETRIC INTERPRETATION

Let 's consider the following mapping between the space of possible worlds and the one of sentences.

$$X \xrightarrow{\Psi} Y$$

$$V=[P(V1,P(V2),P(V3),P(V4)]^t \dashrightarrow \Psi(V) = \Pi = [P(P),P(\neg PvQ),P(Q)]^t.$$

The system $\Psi(V) = \Pi$ is equivalent to:

$$\begin{cases} P(V_1) + P(V_2) = P(P) \\ P(V_1) + P(V_3) + P(V_4) = P(\neg PvQ) \\ P(V_1) + P(V_3) = P(Q) \end{cases}$$

$$\text{s.t. } \sum_{i=1}^{i=4} P(V_i) = 1.$$

By doing some algebraic transformations, one obtains:

$P(\neg PvQ) + P(P) - 1 - P(Q) = P(V_1) + P(V_3) + P(V_4) + P(V_1) + P(V_2) - 1 - P(V_1) - P(V_3) = - P(V_3) \leq 0$. Then, one can write: $P(\neg PvQ) + P(P) -1 \leq P(Q)$.

One also notices that:

$P(Q) - P(\neg PvQ) = P(V_1) + P(V_3) - P(V_1) - P(V_3) - P(V_4) = - P(V_4) \leq 0$.

In conclusion, we obtain:

$P(\neg P \vee Q) + P(P) - 1 \leq P(Q) \leq P(\neg P \vee Q)$.

Remark 3.1. The bounds relative to P(Q) is not found by a geometric approach, but by an algebraic approach.

The geometric interpretation is based on the following theorem.

Theorem 3.2. *X and Y are 2 normed spaces, Ψ a linear mapping from X to Y. If $\{V_i\}$ is a set of extreme vectors according the norm défined in X and Y, then the set $\{\Psi(V_i)\}$ is also a set of extreme vectors according to the same norm.*

Proof. Because Ψ is a linear operator.

4. THE USE OF THE INFINITE NORM : $L_\infty(x) = \text{MAX} \mid x_i \mid$

4.1. The notion of extreme vector

We express in the space X (normed space of possible worlds) the extreme vector concept by the fact that the norm of any vector V is equal to 1, which can be written as $\| V \|_\infty = 1$. Thus, we can write $\text{Max} \mid V_i \mid = 1 \Leftrightarrow \text{Max} \mid P(V_i) \mid = 1$ $\forall i \in \{1,2,3,4\}$. However, since $\sum P(V_i) = 1$, we know that if one component of the vector V is equal to 1 , the others must be 0. Finally, we have obtained 4 extreme vectors (according to $\| . \|_\infty$) corresponding to the base of IR^4 (the classical Euclidian space). By using the linear relation due to the product of the consistent matrix C by the column vector of possible worlds probabilities V, one can write:

$\Psi(V) = \Pi \Leftrightarrow CV = \Pi$.

The extreme vectors in the space of sentences are the column vectors of the matrix C (consistent). Geometrically, the consistent region is the inside of the convex hull described by of the extreme vectors $\Psi(V)$. The points outside this volume are inconsistent. When using the mixed product in order to determine the plane equations, the constraints corresponding to the consistent region are easily found. Thus, we obtain the inequations:

$P(Q) \leq P(\neg P \vee Q)$,

$P(\neg P \vee Q) + P(P) - 1 \leq P(Q)$.

Remark 4.1.1. The bounds relative to P(Q) are now found geometrically. The geometric region has been searched in order to match with the region found algebraically: it is s a particular construction.

5. THE USE OF THE L1 NORM: $L_1(x) = \Sigma \mid x_i \mid$

Using the property of a linear mapping and the continuity concept, we define an overvalue relative to the norm of any vector in the space of possible worlds. The consistent region is contained inside the maximal area assigned to this overvalue.

5.1. The maximal area containing the consistent region

In the case of the L1 norm, the notion of extreme vector in the space of possible worlds is defined by $\| V \|_1 = 1$. Therefore, by this definition, all the vectors defined in the space of possible worlds become extreme. This remark is inherent to the fact that a probability must satisfy the concept of exclusivity and exhaustivity. Additional information is required in order to obtain a square matrix C. One can assert that the maximal area containing the consistent region is:

$D = \{[P(P),P(\neg P \vee Q),P(Q),P(\tau)]^t$ such that $P(P) + P(\neg P \vee Q) + P(Q) + P(\tau) \leq 4\}$
(where t is the transpose vector form).

5.2. The consistent region

In this section, we set up a theorem which enables us to obtain the consistent region according to this norm.

Proposition 5.2.1. *Any vector defined in the space of possible worlds is extreme according to this norm.*

Proof. This is due to the fact that any probability vector must satisfy $\Sigma P(V_i) = 1$.

<u>The technical aspect</u>

Before giving the method and a solution, we have to present some topological properties. Let $P = \{V= [P(V_1),P(V_2),...,P(V_n)]^t \ / \ \Sigma P(V_i) = 1$ and $\forall i, P(V_i) = P_i \geq 0\}$.

Lemma 5.2.2. *P is the space of possible worlds and it is a convex and compact set in IR^n.*

Proof. P is convex and closed, because it is the intersection of closed convex sets; it is also bounded, because it is contained inside the unity sphere $\| V \|_1 = 1$.

Theorem 5.2.3. *Let U be a linear operator in IR^n, $C = (C_{ij})$ its associated matrix relative to the classical base of IR^n. If $\Sigma C_{ij} = n$, $\forall j$ (n - 1 is the number of sentences in the set of beliefs), $C_{ij} \geq 0$, then one can write the following assertions:*

-1- $U(P) \subset nP$
-2- $\| \Pi \|_1 = n \| V \|_1$.

Proof. By using the hypothesis of this theorem, one can write:

$$\| CV \|_1 = \sum_{i=1}^{i=n}\sum_{j=1}^{j=n} C_{ij}V_j = \sum_{j=1}^{j=n}\sum_{i=1}^{i=n} C_{ij}V_j = \sum_{j=1}^{j=n} V_j \sum_{i=1}^{i=n} C_{ij} = n \sum_{j=1}^{j=n} V_j$$

and both of the relations 1 and 2 are proved.

Proposition 5.2.4. *The sets P, $U(P)/n$, $U^2(P)/n^2$, ..., $U^k(P)/n^k$,..., form a decreasing sequence with an intersection D different of $\varnothing$.*

Proof. $U^k(P)/n^k$ is a decreasing sequence when applying the theorem 5.2.3., so any finite intersection of the sequence is different of $\varnothing$. Now, for all k, U^k is continuous, because it is a linear operator in a finite dimension space, thus, $U^k(P)/n^k$ is compact, and $\cap U^k(P)/n^k = D \neq \varnothing$.

Proposition 5.2.5. *$U(D) = D$ and for each probability vector V, $U(V) \in D$.*

Proof. If $V \in D$, for all $k \geq 0$, $V \in U^k(P)/n^k$, therefore $U(V) \in U^{k+1}(P)/n^k$ e.g., $U(V) \in D$. If $W \in D$, by definition of D, for all $k \geq 0$, $\exists V_k \in U^k(P)/n^k$ such that $W = U(V_k)$. As P is compact, the sequence V_k has at least one adherence value V, then $V \in D$, since $V_k \in \cap U^i(P)/n^i$ with $i \leq k$ and the $U^i(P)/n^i$ are closed and decreasing. In conclusion, as $U(D) \subset D$ and $D \subset U(D)$ then $U(D) = D$.

Theorem 5.2.6. *If $C(k) = (C_{ij}(k))$ is the matrix associated to U^k/n^k relative to the canonical base of IR^n, then a strictly increasing sequence $k_1, k_2, k_3,...,$ of natural numbers exists so that, for all i and j, the sequence $(C_{ij}(k_p))$ tends to the limit matrix $\beta = (\beta_{ij})$ when p tends to infinity and $L(P) = D$ where L is a linear operator associated to the matrix $\beta = (\beta_{ij})$.*

Proof. For all (i,j,k), we can write: $0 \leq C_{ij}^k \leq 1/n^k$; so, for all (i,j) the sequence (C_{ij}^k) possesses an adherence value (Bolzano-Wiertrass). So by setting in order the finite set of indices i and j, we can by n^2 extractions of the sequence find an increasing sequence of natural numbers (k_p) so that, for all i and j, the sequence $(C_{ij}(k_p))$ tends to $\beta = (\beta_{ij})$ (adherence value) when p tends to infinity. For all $p \geq 1$, we can write using the definition of L: $L(P) \subset U^{kp}(P)/n^{kp}$, thus, also $L(P) \subset D$ because $U^i(P)/n^i$ is a decreasing sequence. Let $V \in D$; for all natural number p, we can find $V_{kp} \in P$ such that $V = U^{kp}(V_{kp})/n^{kp}$. If W is an adherence value associated to the sequence V_{kp}, we obtain the result by using for example the following inequality:

$\| U^{kp}(V_{kp})/n^{kp} - L(W)\| \leq \| U^{kp}(V_{kp})/n^{kp} - L(V_{kp}) \| + \| L(V_{kp}) - L(W) \|.$

Proposition 5.2.7. *P is the convex hull relative to $\{e_i\}$, D is the convex hull relative to $\{L(e_i)\}$, where $\{e_i\}$ is the canonical base associated to IR^n.*

Proof. This is due to the fact that L is a linear operator and L(P) = D.

We can find the application of the previous topological results in [4].

6. THE L1 ISOMETRIES

We are interested by special automorphisms called the isometries.

Definition 6.1. *f: X ------> Y a non singular mapping, X and Y are 2 normed spaces, f is called an isometry if:*

$\| x - y \| = \| f(x) - f(y) \| \quad \forall (x,y) \in X*Y.$

As each vector defined in the space of possible worlds is extreme according to this norm, when dealing with isometries, we obtain the following proposition.

Proposition 6.2. *If $C \in L(IR^n)$, the following assertions are equivalent:*

-1- $\| C*V \|_1 = \| \Pi \|_1 = \| V \|_1 = 1$

-2- $\sum C_{ij} = 1 \quad \forall j \in [1..n]$ *and* $C_{ij} \geq 0.$

Proof. it is the theorem 5.2.3. for n = 1.

Remark 6.3. This is a particular case (n = 1) of the work done previously. However, any vector $\Pi = (\Pi_i)$ satisfies $\sum \Pi_i = 1$. This case is the one where the space of sentences is a probability space. The solution for P(Q) is unique and is equal to 1- P(¬P∨Q) - P(P) - P(A).

Proposition 6.4. *One must change the classical representation of the truth values and takes $\varepsilon 1$ and $\varepsilon 2$ with $(0 \leq \varepsilon 1, \varepsilon 2 < 1)$ for new representations. If the number of predicates increases, we have to choose very small values for $\varepsilon 1$ and $\varepsilon 2$ in the interval [0..1].*

Proof. This is due to the equation 2 of the proposition 6.2.

7. THE USE OF THE L2 NORM: $L_2(x) = (\sum x_i^2)^{1/2}$

In this section, we use the Euclidean norm to find the minimal sphere containing the "consistent" region. We show that the problem is equivalent to

the research of eigenvalues associated to a quadratic form.

7.1. The minimal sphere containing the consistent region

This approach is based on weights W_i given to the sentences and possible worlds instead of probabilities and the concept of continuity in the space of finite dimensions. One condition is that each weight belongs to the interval [0..1] and their sum can be greater than 1. By using this norm, we are approaching a fuzzy logic [5]. In order to make calculations easier, one can add additional information τ (tautology) to fulfil the necessary hypotheses which enable us to apply some methods using spectral theory. We impose that the extreme vector is defined according to this norm as:

$$\| V \|_2^2 = 1 \Leftrightarrow \sum W_i^2 = 1.$$

Our interest is to determine the minimal sphere containing the consistent region. The constraint corresponding to this aim is: $\exists k_0 \in IR^+$ such that:

$$\| \Pi \|_2^2 = k_0. \tag{1}$$

$$\exists\, K>0 \,;\, \| C^*V \|_2 \leq K \| V \|_2 \Leftrightarrow \| \Pi \|_2 \leq K \tag{2}$$

The equation (1) is equivalent to the sphere whose equation in IR^4 is:

$$W^2(P) + W^2(\neg P v Q) + W^2(Q) + W^2(\tau) = k_0.$$

We choose k_0 equals to Inf{K_i} where K_i is the set of positive numbers satisfying the relation (2). The additional information τ is a tautology added as the last row vector in the consistent matrix. Finally, as $C^*V = \Pi$, our optimisation approach is to solve according to $W(V_i) = w_i$ the following problem:

$$\text{Max } f(w_1,w_2,w_3,w_4) =$$
$$\text{Max}\{(w_3+w_4)^2+(w_1+w_2+w_4)^2+(w_2+w_4)^2+(w_1+w_2+w_3+w_4)^2\}$$
$$\text{s.t. } Q(w_1,w_2,w_3,w_4) = \sum w_i^{\,2} = 1 \text{ with } i \in \{1,2,3,4\}.$$

The mapping Q is quadratic and positive definite and therefore solutions can be found by using spectral theory properties. One can transform our optimized problem into an equivalent one. In order to obtain a well known function to optimize, we shall change the variables in the following manner:

$$(w_3+w_4) = x_1 \,;\; (w_1+w_2+w_4) = x_2 \,;\; (w_2+w_4) = x_3 \,;\; (w_1+w_2+w_3+w_4) = x_4.$$

The determinant associated to the previous linear system is $\neq 0$ (is equal to 1 in this example) thus, the solution for w_i is unique. The problem is transformed into the following one:

$\text{Max } f(x_1,x_2,x_3,x_4) = \text{Max } \sum x_i^2$
s.t. $Q(x_1,x_2,x_3,x_4) = (x_2-x_3)^2 + (x_4+x_3-x_1-x_2)^2 + (x_4-x_2)^2 + (x_1+x_2-x_4)^2 = 1.$

<u>The technical aspect</u>

Let's consider IR^4 with its Euclidian structure, we associate to the quadratic form Q, for example the symmetric automorphism U defined by:
$Q(x) = (U(x),x) = (x,U(x))$, the comma means the scalar product. By using this automorphism, the problem can be less complicated to solve when using the spectral theory. Therefore, the automorphism which can be associated to the quadratic form Q is not unique.

The differential of g: $IR^4 \longmapsto IR$
$x \longmapsto Q(x) - 1$
is the following mapping:
$h \longmapsto Q'(x) . h = 2(U(x),h).$

The differential of f: $IR^4 \longmapsto IR$
$x \longmapsto (x,x)$
is the following mapping:
$h \longmapsto f'(x).h = 2(x,h).$

Proposition 7.1.1. *Let f,g : D ----------> IR differentiable in the open D of E (vector space), let $V = \{x \in D \mid g(x) = 0\}$ if $f \mid V$ possesses an extremum for x_0, then the forms $f'(x_0)$ and $g'(x_0)$ are proportional.*

Proof. see [3].

Proposition 7.1.2. *If $E = IR^P$, this proportionality is expressed by the proportionality of partial derivatives.*

Proof. Since $f'(x_0)$ and $g'(x_0)$ are proportional, then it is easy to have the proportionality of the partial derivatives. The proposition 7.1.1. proves that: $\exists \lambda \in IR$ such that $(U(x),h) = \lambda(x,h)\ \forall h$. Thus, by the proposition 7.1.2., we deduce: $\exists\ \lambda \in IR$ with: $U(x) = \lambda x$. The eigenvalues are all positive (Q is positive definite), the extrem vectors are obtained in all points x which are eigenvectors of the automorphism U with $Q(x) = 1$. In those solution points for example x_0, one can write:

$f(x_0 + h) = (x_0,x_0) + 2(x_0,h) + (h,h)$
$Q(x_0 + h) = Q(x_0) + 2(U(x_0),h) + Q(h)$
with: $Q(x_0) = Q(x_0 + h) = 1.$
Thus, we obtain the following relations:
$(x_0,h) = (U(x_0),h)*1/\lambda = - Q(h)/2\lambda$ and thus we can write:

$f(x_0+h) = f(x_0) + (h,h) - Q(h)/\lambda$.

Since λ_m and λ_M are respectively the minimal and maximal eigenvalue, we have: $(\forall h)\ \lambda_m(h,h) \leq Q(h) \leq \lambda_M(h,h)$; for $\lambda = \lambda_m$ we obtain:

$(\forall h)\ f(x_0 + h) \leq f(x_0)$ and thus the fonction f possesses a maximum, and this minimal eigenvalue is the solution to the problem.

So $\lambda = \lambda_m = k_0 = \text{Max } f(x_1,x_2,x_3,x_4)$

for $\lambda = \lambda_M$, we obtain a minimum.

In any point x_0 obtained, $(x_0,x_0) = (U(x_0)/\lambda,x_0) = 1/\lambda\ Q(x_0) = 1/\lambda$.

Many numerical algorithms exist [6] for determining the minimal eigenvalue. As it has been proved, this minimum is equal to $k_0 \leq 1$. The consistent region is limited by the positive part of the sphere in which the radius is equal to $(\text{Inf } \lambda_i)^{1/2}$.

8. THE L2 ISOMETRIES

The idea has to do with the following question: in what conditions could the consistent region be contained inside the unity ball, which is the unity sphere in 3-dimensions? Mathematically this is expressed by the following equality:

$$\| C^*V \|_2^2 = \| V \|_2^2 = 1 \iff \| \Pi \|_2^2 = 1.$$

Proposition 8.1. *If $C \in L(IR^n)$ the following assertions are equivalent:*

-1- C is an orthogonal matrix
-2- $\| CV \|_2 = \| V \|_2$ (C is called an isometry).

Proof. $\| CV \|_2^2 = (CV,CV) = (V,C^*CV)$ as $C^*C = Id$ (C orthonormal matrix) thus:

$\| CV \|_2^2 = (V,V) = \| V \|^2$ and $\| CV \|_2 = \| V \|_2$.

Proposition 8.2. *If $C \in L(IR^n)$ is an isometry then C^t(obtained by transposing rows and columns) remains an isometry. The set of all isometries is a group for the product operation.*

Proof. $(C^*C)^t = Id = C^t(C^*)^t = C^t(C^t)^* = Id$; this proves that C^t is an orthogonal matrix which is equivalent to an isometry.

In the general case, the following kind of matrices are isometries:

$$
\mathcal{C} = \begin{array}{c}
\begin{array}{cccccccc} & & & p & & & q & \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & 0 & \dots & & 0\dots0\dots0 \\
0 & 1 & 0 & 0 & \dots & 0 & \dots0\dots\dots0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0\dots & \dots & \dots0 & \cos\theta & \dots & \sin\theta & \dots0\dots & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0\dots & 0\dots & \dots & -\sin\theta & \dots & \cos\theta & \dots & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0\dots & 0\dots & 0\dots & \dots & \dots & \dots & \dots & 1
\end{bmatrix}
\end{array}
$$

where p and q are subject to the condition: $1 \leq p < q \leq n$, p and q are not only indications of columns but indications of lines as well, n is the dimension of the matrix, q is the angle of the vector of indice p and q of the rotation.

Proposition 8.3. *The orthogonal matrices require a multivalued logic (trivalued logic (0,1,θ)). The classical binary logic is not suitable when using the L2 norm.*

Proof. This is due to the form of the matrix $\mathcal{C}$.

9. CONCLUSION

With the changes of topologies, one found out that it is possible to transform the space of predicates into a probability space. The condition of L1 isometry is equivalent to exclusivity and exhaustivity of the predicates. The determination of the probability associated to a sentence S given the other sentences is then unique. Some of L2 isometries have been cited, they also require different representations of the truth values. Their minimal area containing the consistent region is the unity sphere. However, some works need to be done in order to determine explicitly the values of the new representations.

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