Lecture Notes in Statistics 89

P. Cheeseman
R. W. Oldford (Eds.)

Selecting Models from Data

Artificial Intelligence and Statistics IV

Springer-Verlag

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Capturing observations in a nonstationary hidden Markov model

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ABSTRACT This paper is concerned with the problem of morphological ambiguities using a Markov process. The problem here is to estimate interferent solutions that might be derived from a morphological analysis. We start by using a Markov chain with one long sequence of transitions. In this model the states are the morphological features and a sequence correponds to a transition from one feature to another. After having observed an inadequacy of this model, one will explore a nonstationary hidden Markov process. Among the main advantages of this latter model we have the possibility to assign a type to a text, given some training samples. Therefore, a recognition of "style" or a creation of a new one might be developped.

27.1 Introduction

27.1.1 Automatic analysis of natural language

This work lies within a textual analysis system in natural language discourse (French in our case). In most systems used today, the analysis process is divided into *levels*, starting from morphology (first level) through syntax, and semantics to pragmatics. These levels are sequentially activated, without backtracking, originating in the morphological phase and ending in the pragmatic one. Therefore, the i-th level knows only the results of preceding levels. This means that, at the morphological level, each word in the text (*a form*) is analyzed autonomously out of context. Hence, for each form, one is obliged to consider all possible analysis.

Example : let's consider the sequence ot the two forms *cut* and *down* :

- *cut* can be given 3 analyses : verb, noun, adjective ;
- *down* can be a verb, an adverb or a noun.

The number of possible combinations based upon the independance of the analysis of one form in relation with the others implies that the phrase *cut down* is liable to *nine* interpretations, independently on the context.

These multiple solutions are transmitted to syntactic parsing which doesn't eliminate them either. In fact, as a syntactic parser generates its own interferent analyses, often from interferent morphology analysis, the problems with which we are confronted are far from being solved. In order to provide a solution to these problems, we have recourse to statistical methods. Thus the result of the morphological analysis is filtered when using a Markov model.

[1] *Selecting Models from Data: AI and Statistics IV.* Edited by P. Cheeseman and R.W. Oldford.

27.1.2 *Morphological analysis*

A morphological analyser must be able to cut up a word form into smaller components and to interpret this action. The easiest segmentation of a word form consists in separating word terminations (inflexional endings) from the rest of the word form called *basis*. We have then got a *inflexional morphology*. A more accurate cutting up consists in splitting up the basis into affixes (*prefixes, suffixes*) and *root*. This is then called *derivational morphology*.

The interpretation consists in associating the segmentation of a word form with a set of informations, particulary including :

- the general morphological class : verb, noun-adjective, preposition, ...
- the values of relevant morphological variables : number, gender, tense, ...

Therefore, an interpretation is a class plus values of variables ; such a combination is called a *feature*. Note that a word form is associated with several features in case where there are multiple solutions.

27.1.3 *Why statistical procedures?*

Because of the independance of the analysis levels, it is difficult to provide contextual linguistic rules. This is one of the reasons why we fall back on statistical methods. These latter method possess another advantage : they reflect simultaneously language properties, e.g. the impossibility to obtain a determinant followed directly by a verb, and properties of the analysed corpus, e.g. a large number of nominal phrases.

Some researchers used Bayesian approaches to solve the problem of morphological ambiguities. However, these methods have a clear conceptual framework and powerful representations, but must still be knowledge- engineered, rather than trained. Very often in the application of these methods, researchers have a good observation of the individuals of the population, *because the observation is a relative notion*. Therefore, we have difficulty in observing possible transitions of the individuals. The way of "capturing" the individuals depends on the environment encountered.

27.2 A morphological features Markov chain

27.2.1 *The semantic of the model*

Let m be the number of states, T the length of state sequence and $\{f_i/1 \leq i \leq m\}$ the states or morphological features ; we have only one individual ($n = 1$) for each transition time $t = 1, 2, \ldots, T$. A first order m-states Markov chain is defined by an $m \times m$ state transition matrix P, an $m \times 1$ initial probability vector Π, where :

$$P = (P_{f_i,f_j})$$

$$i, j = 1, 2, \ldots, m$$

$$P_{f_i,f_j} = Prob[e_{t+1} = f_j / e_t = f_i]$$

$$\Pi_{f_i} = Prob[e_1 = f_i]$$

$$i = 1, 2, \ldots, m$$

By definition, we have :

$$\sum_{j=1}^{j=m} P_{f_i,f_j} = 1 \qquad pour\ i = 1, 2, \ldots, m$$

$$\sum_{k=1}^{k=m} \Pi_{f_k} = 1$$

The probability associated to a realization E of this Markov chain is :

$$Prob[E/P, \Pi] = \Pi_{e_1} \times \prod_{t=2}^{t=T} P_{e_{t-1},e_t}$$

27.2.2 *Estimation of transition probabilities*

As pointed out by Bartlett in Anderson and Goodman [AG57] the asymptotic theory must be considered with respect to the variable *number of times of observing the word form in a single sequence of transitions*, instead of the variable *number of individuals in a state when T is fixed.* However, this asymptotic theory was considered because the number of times of observing the word form increases ($T \to +\infty$). Furthermore, we cannot investigate the stationary properties of the Markov process, since we only have one word form (one individual) at each transition time. Therefore, we assumed stationarity. Thus, if N_{f_i,f_j} is the number of times that the observed word form was in the feature f_i at time $t-1$ and in the feature f_j at time t, for $t \in \{1, 2, \ldots, T\}$, then the estimates of the transition probabilities are :

$$\hat{P}_{f_i,f_j} = \frac{N_{f_i,f_j}}{N_{f_i+}}$$

where N_{f_i+} is the number of times that the word form was in state f_i. The estimated transition probabilities are evaluated on one training sample. We removed the morphological ambiguities by choosing the sequence E of higher probability.

27.3 A Markov model with hidden states and observations

The inadequacy of the previous model to remove certain morphological ambiguities has led us to believe that some unkown hidden states govern the distribution of the morphological features. Instead of passing from one morphological feature to another, we focused only on the surface of one random sample, i.e. an observation was a morphological feature. As pointed out in [ROU88], this latter entity cannot be extracted without a context effect in a sample. In order to consider this context effect, we have chosen criteria like *the nature of the feature, its successor feature, its position in a sentence, the position of the sentence in the text.* An observation o_i is then a *known hidden vector* whose components are values of the criteria presented here. Of course, one can explore other criteria.

Définition 1 *A hidden Markov model (HMM) is a Markov chain whose states cannot be observed directly but only through a sequence of observation vectors.*

A HMM is represented by the state transition probability P, the initial state probability vector Π and a $T \times K$ matrix V (K is the number of states) ; the elements of V are the conditional densities $v_i(o_t) = \textit{density of observation } o_t \textit{ given } e_t = i$. Our aim is the determination of the optimal model estimate $\mathcal{V}^* = (\Pi^*, P^*, V^*)$ given a certain number of samples : this is the *training problem.*

Theoreme 1 *The probability of a sample $S = \{o_1, o_2, \ldots, o_T\}$ given a model $\mathcal{V}$ can be written as :*

$$Prob(S/\mathcal{V}) = \sum_{E} \Pi_{e_1} v_{e_1}(o_1) \times \prod_{t=2}^{t=T} P_{e_{t-1},e_t} v_{e_t}(o_t)$$

Proof : For a fixed state sequence $E = (e_1, e_2, \ldots, e_T)$, the probability of the observation sequence $S = \{o_1, o_2, \ldots, o_T\}$ is :

$$Prob(S/E, \mathcal{V}) = v_{e_1}(o_1) \times v_{e_2}(o_2) \times \ldots \times v_{e_T}(o_T)$$

The probability of a state sequence is :

$$Prob(E/\mathcal{V}) = \Pi_{e_1} \times P_{e_1,e_2} \times P_{e_2,e_3} \times \ldots \times P_{e_{T-1},e_T}$$

Using the formula :

$$Prob(S, E/\mathcal{V}) = Prob(S/E, \mathcal{V}) \times Prob(E/\mathcal{V})$$

and summing this joint probability over all possible states sequences E, one demontrates the theorem.

The interpretation of the previous equation is : initially at time $t = 1$, the system is in state e_1 with probability Π_1 and we observe o_1 with probability $v_{e_1}(o_1)$. The system then makes a transition to state e_2 with probability P_{e_1,e_2} and we observe o_2 with probability $v_{e_2}(o_2)$. This process continues until the last transition from state e_{T-1} to state e_T with probability P_{e_{T-1},e_T} and then we observe o_T with probability $v_{e_T}(o_T)$.

In order to determine one of the estimate of the model $\mathcal{V} = (\Pi, P, V)$, one can use the maximum likehood criterionn (or a max entropy) for a certain family S_i where $i \in \{1, 2, \ldots, L\}$ of training samples. Some methods of choosing representative samples of fixed length are presented in [BOU92]. The problem is expressed mathematically as :

$$\max_{v_i} f(S_1, S_2, \ldots, S_L/\mathcal{V}) = \max_{v_i} \{ \prod_{j=1}^{j=L} [\sum_{E} \Pi_{e_1} \times v_{e_1}(o_1^j) \times \prod_{t=2}^{t=T} P_{e_{t-1},e_t} v_{e_t}(o_t^j)] \}$$

There is no known method to solve this problem analytically, that is the reason why we use iterative procedures. We start by determining first the optimal path for each sample. An optimal path E^* is the one which is associated to the higher probability of the sample. Using the well-known Viterbi algorithm, one can determine this optimal path. The different steps for finding the single best state sequence in the Viterbi algorithm are :

step 1 : initialization

$$\delta_1(i) = \Pi_i v_i(o_1) \qquad (1 \leq i \leq K)$$

$$\psi_1(i) = 0$$

step 2 : recursion for $2 > t > T$ and $1 \leq j \leq K$:

$$\delta_t(j) = \max_{1 \leq i \leq K}[\delta_{t-1}(i)P_{i,j}]v_j(o_t)$$

$$\psi_t(j) = \arg\max_{1 \leq i \leq K}[\delta_{t-1}(i)P_{i,j}]$$

step 3 : termination

$$P^* = \max_{1 \leq i \leq K}[\delta_T(i)]$$

$$e_T^* = \arg\max_{1 \leq i \leq K}[\delta_T(i)]$$

state sequence backtracking for $t = T-1, T-2, \ldots, 1$:

$$e_T^* = \psi_{t+1}e_{t+1}^*$$

P^* is the state-optimized likelihood function and $E^* = \{e_1^*, e_2^*, \ldots, e_T^*\}$ is the optimal state sequence. Instead of tracking all possible paths, one successively tracks only the optimal paths E_i^* of all samples. Thus, this can be written as :

$$g(o_1, o_2, \ldots, o_T; E^*, \mathcal{V}) = \max_E\{\Pi_{e_1} \times v_{e_1}(o_1) \times \prod_{t=2}^{t=T} P_{e_{t-1},e_t}v_{e_t}(o_t)\}$$

This computation has to be done for all the samples. Among all the v_i ($i \in \{1, 2, \ldots, L\}$) associated to optimal paths, we decide to choose as best model estimate the one which maximizes the probability associated to a sample. It can be written as :

$$\mathcal{V}^* = arg\{\max_{v_i} g(o_1^i, o_2^i, \ldots, o_T^i; E^*, \mathcal{V}_i)$$

$$i \in \{1, 2, \ldots, L\}$$

27.4 The different steps of the method

We present an interactive method which enables us to obtain an estimator of the model V. This method is suitable for direct computation.

First step : one has to cluster the sample with respect to the chosen criteria. two possibilities are offered : a *classification* or a *segmentation*. In this latter procedure, the user may structure the states ; operating in this way, the states appear like unknown hidden states. However, in a classification the system structures its own states according to a suitable norm. Thus, the states appear like unkown hidden ones. The clusters formed by one of the two procedures represent the first states of the model, they form *the first training path.*

Second step : one estimate the transition probabilities using the following equations and the probability of each training vector for each state $v_i(o_t)$. This is the first model $\mathcal{V}_1$. Let $i, i \in \{1, 2, \ldots, K\}$ and $t \in \{1, 2, \ldots, T\}$.

- Let $Nb(o_1, i)$ be the number of times the observation o_1 belongs to the state i and Nbp the number of training paths, then :

$$\hat{\Pi}_i = \frac{Nb(o_1, i)}{Nbp}$$

- Let $Nb(o_{t-1}, i; o_t, j)$ be the number of times the observation o_{t-1} belongs to the state i and the observation o_t belongs to the state j , then :

$$\hat{P}_{i,j}(t) = \frac{Nb(o_{t-1}, i; o_t, j)}{Nb(o_{t-1}, i)}$$

- The previous estimation formula can be written as :

$$\hat{P}_{i,j}(t) = \frac{N_{i,j}(t)}{N_i(t-1)} = \frac{N_{i,j}(t)}{N_{i+}(t)}$$

where $N_{i,j}(t)$ is the number of transitions from state i at time $t-1$ to state j at time t and $N_i(t-1)$ the number of times the state i is visited at time $t-1$.

- Let $Nbex(o_1, i)$ be the expected number of times of being in state i and observing o_t and $Nbex(i)$ the expected number of times of being in state i, then :

$$\hat{v}_i(o_t) = \frac{Nbex(o_1, i)}{Nbex(i)}$$

Third step : one computes $f(o_1, o_2, \ldots, o_T; \mathcal{V}_1)$ and determines the next training path, or clustering, necessary to increase $f(o_1, o_2, \ldots, o_T; \mathcal{V}_1)$. We apply the second step to this training path. The procedure is repeated until we reach the maximum value of the previous function. At this optimal value, we have E_1^* and v_1 of the first sample. This step uses Viterbi algorithm.

This algorithm is applied to a family of samples of the same text, so we obtain a family of E_i^* and $\mathcal{V}_i$. As mentioned previously, one decides reasonably to choose the model $\mathcal{V}^*$ whose probablitiy associated to a sample is maximum. This last model makes the sample the most representative, i.e. *we have a good observation in some sense*. This optimal model estimate is considered as *a type of the text processed.*

27.5 Test for first-order stationarity

As outlined by Anderson ang Goodman [AG57] the following test can be used to determine whether the Markov chain is first-order stationary, or not. Thus, we have to test the null hypothesis (H) :

$$P_{i,j}(t) = P_{i,j} \qquad (t = \{1, 2, \ldots, T\}$$

The likehood ratio with respect to the null and alternate hypothesis is :

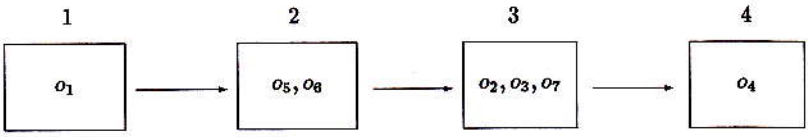
$$\lambda = \prod_{t=1}^{t=T} \prod_{i=1}^{i=K} \prod_{j=1}^{j=K} \frac{P_{i,j}^{N_{i,j}(t)}}{P_{i,j}^{N_{i,j}(t)}(t)}$$

We now determine the confidence region of the test. In fact, the expression $-2\log\lambda$ is distributed as a Chi-square distribution with $(T-1) \times K \times (K-1)$ degrees of freedom when the null hypothesis is true. As the distribution of the statistic $S = -2\log\lambda$ is χ_2, one can compute a β point ($\beta = 95, 99.95$ %, etc.) as the threshold S_β. The test is formulated as :
If $S < S_\beta$, the null hypothesis is accepted, i.e. the Markov chain is first-order stationary. Otherwise, the null hypothesis is rejected at $100\% - \beta$ level of signifiance, i.e. the chain is not a first order stationary and one decides in favour of the nonstationary model.

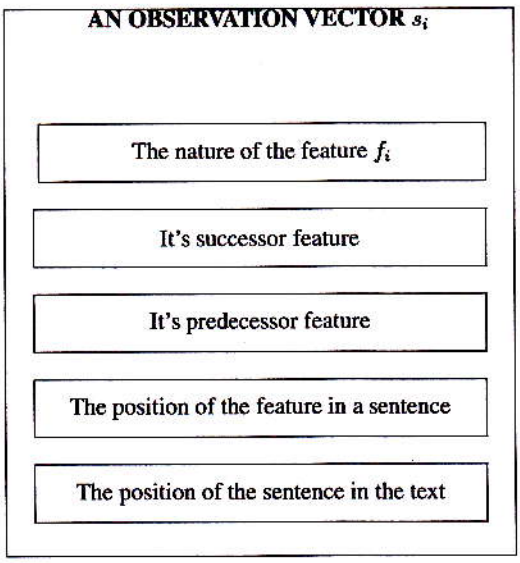
27.6 How to solve the morphological ambiguities

This is the most important phase of our application. Let's consider an example of nine possible paths encountered in a test. Among these paths, the system has to choose the most likely according to the probability measure (see the third figure). Our decision of choosing the most likely path comes from the optimal model $\mathcal{V}^*$ obtained in the training phase. We show in this example how to remove the morphological ambiguities.

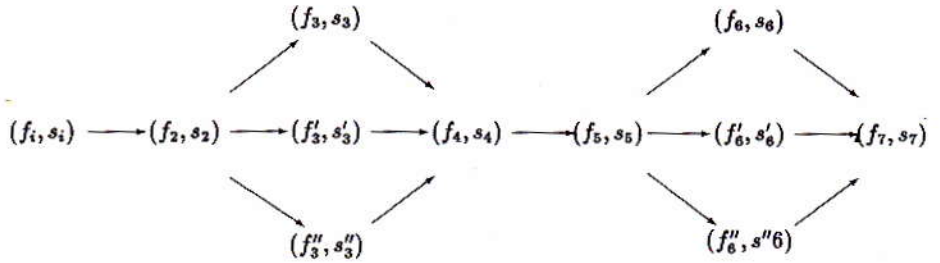
If the optimal state sequence obtained in the training phase is the one which corresponds to the figure :



K=4 : the optimal state sequence is $1, 3, 3, 4, 2, 2, 3$



the one for example can choose between the two following paths of the figure :



$$\begin{array}{lccccccc} Path\,1 & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \\ Path\,2 & s_1 & s_2 & s'_3 & s_4 & s_5 & s'_6 & s_7 \end{array}$$

One compute the probabilities of these two realizations of the observations o_i $(i = 1, 2, \ldots, 7)$ using the formula :

$$Prob(o_1, o_2, \ldots, o_7/\mathcal{V}^*) = \Pi_{e_1 v_{e_1}}(o_1) \times \prod_{t=2}^{t=7} P_{e_{t-1}, e_t} v_{e_t}(o_t)$$

The first figure shows that each s_i belongs to a state e_1 and, using the optimal model $\mathcal{V}^* = (\Pi, P, V)$ one can compute the probability of a path. Our decision to remove the morphological ambiguities is to choose the path with the highest probability.

27.7 Conclusion

We have presented a new approach for solving the morphological ambiguities using a hidden Markov model. This method may also be applied to other analysis levels as syntax. The main advantage of the method is the possibility to assign many different classes of criteria (fuzzy or completely known) to the training vectors and investigating many samples. Furthermore, we can define a "distance" between any sample and a family of type of texts called models. One can choose the model which gives the higher probability of this sample and conclude that the sample belongs to the specific type of texts. We can also develop a proximity measure between two models $\mathcal{V}_1^*$ and $\mathcal{V}_2^*$ through representative samples. However, some precautions must be taken in the choice of the distance used between the training vectors in the cluster process. In fact, the value of the probability associated to a sample may depend on this norm and, therefore, the choice of the best model estimated can be affected.

So far, we supposed that the criteria described the observations and the states are completely known (hard observation). Very often, when we want to make deep investigations, fuzziness or uncertainty due to some criteria or states are encountered, what should be done in this case? How can we cluster the observations according to some uncertainty measure? What is the optimal path and the best estimate model according to the uncertainty meausure? We are working in order to propose solutions to those questions in the case of a probabilistic [BOU91,BOU] and fuzzy logic.

27.8 References

[AG57] T.W. ANDERSON and L. A. GOODMAN. Statistical inference about markov chains.

Annals of Mathematical Statistics, 28, 1957.

[BOU] D. BOUCHAFFRA. *A relation between isometrics and the relative consistency concept in probabilistic logic.* Elsevier, North Holland.

[BOU91] D. BOUCHAFFRA. A relation between isometrics and the relative consistency concept in probabilistic logic. In *13th IMACS World Congress on Computational and Applied Mathematics*, juillet 1991.

[BOU92] D. BOUCHAFFRA. *Echantillonnage multivarié de textes pour les processus de Markov et introduction au raisonnement incertain dans le Traitement Automatique de la Langue naturelle.* PhD thesis, Université des Sciences Sociales, novembre 1992.

[ROU88] J. ROUAULT. *Linguistique automatique : applications documentaires.* P. Lang, 1988.