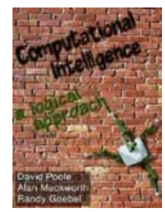
Chapter 10: Using Uncertain Knowledge

- **Introduction**
- **Probability**



D. Poole, A. Mackworth, and R. Goebel, *Computational Intelligence: A Logical Approach*, Oxford University Press, January 1998

Introduction

- Agents don’t have complete knowledge about the world.
- Agents need to make decisions based on their uncertainty.
- It isn’t enough to assume what the world is like.
  (not omniscient but take decision in uncertain world!)
- Example: wearing a seat belt.
  (not accident or accident entails not seat belt!)
- An agent needs to reason about its uncertainty.
- When an agent makes an action under uncertainty it is gambling ⇒ probability.

Probability: Frequentists vs. Subjectivists

- **Probability is an agent's measure of belief in some proposition — subjective probability = measure of belief of an agent given its knowledge! It is adopted**
- **Example: Your probability of a bird flying is your measure of belief in the flying ability of an individual based only on the knowledge that the individual is a bird.**
  - **Other agents may have different probabilities, as they may have had different experiences with birds or different knowledge about this particular bird.**
  - **An agent's belief in a bird's flying ability is affected by what the agent knows about that bird.**

  (uncertainty is epistemological rather than ontological!)

Numerical Measures of Belief

- **Belief in proposition, *f*, can be measured in terms of a number between 0 and 1 — this is the probability of *f*.**
  - **The probability *f* is 0 means that *f* is believed to be definitely false.**
  - **The probability *f* is 1 means that *f* is believed to be definitely true.**
- **Using 0 and 1 is purely a convention.**
- ***f* has a probability between 0 and 1, doesn't mean *f* is true to some degree, but means you are ignorant of its truth value. Probability is a measure of your ignorance (in the subjective sense!).**

Random Variables

- **A random variable is a term in a language that can take one of a number of different values.**
- **The domain of a variable *x*, written *dom(x)*, is the set of values *x* can take.**
- **A tuple of random variables $\langle x_1, \ldots, x_n \rangle$ is a complex random variable with domain $dom(x_1) * \ldots * dom(x_n)$.**
- **Assignment $x = v$ means variable *x* has value *v*.**
- **A proposition is a Boolean formula made from assignments of values to variables.**

Possible World Semantics

- **A possible world specifies an assignment of one value to each random variable.**
- **$w \models x = v$ means variable x is assigned value v in world w** ($w \models f$: f true in world w)
- **Logical connectives have their standard meaning:**
  - $w \models \alpha \wedge \beta$ **if** $w \models \alpha$ **and** $w \models \beta$
  - $w \models \alpha \vee \beta$ **if** $w \models \alpha$ **or** $w \models \beta$
  - $w \models \neg\alpha$ **if** $w \not\models \alpha$

Semantics of Probability

- **For a finite number of variables with finite domains:**
  - **Define a nonnegative measure *$\mu(w)$* for each world w so that the measures of the possible worlds sum to 1.**

    The measure specifies how much you think the world *w* is like the real world.
  - **The probability of proposition *f* is defined by:**

$$P(f) = \sum_{\omega \models f} \mu(\omega)$$

Axioms of Probability

- **Four axioms define what follows from a set of probabilities:**
  - **Axiom 1 $P(f) = P(g)$ if $f \leftrightarrow g$ is a tautology. That is, logically equivalent formulae have the same probability.**
  - **Axiom 2 $0 \leq P(f)$ for any formula f .**
  - **Axiom 3 $P(\tau) = 1$ if τ is a tautology.**
  - **Axiom 4 $P(f \vee g) = P(f)+P(g)$ if $\neg(f \wedge g)$ is a tautology.**
- **These axioms are sound and complete with respect to the semantics.**

Conditioning

- **Probabilistic conditioning specifies how to revise beliefs based on new information.**
- **You build a probabilistic model taking all background information into account. This gives the prior probability.**
- **All other information must be conditioned on.**
- **If evidence e is the all of the information obtained subsequently, the conditional probability P(h|e) of h given e is the posterior probability of h.**
  (medical diagnosis: e = symptoms and h = diseases, priors = p(diseases) do not see the patients)
- **P(e→f) ≠ P(f | e)**
  birds are rare, non-flying birds are small proportion of birds:
  (P(¬flies | birds) ≠ P(birds → ¬flies)

Semantics of Conditional Probability

- **Evidence e rules out possible worlds incompatible with e.**
- **Evidence e induces a new measure, e, over possible worlds**

$$\mu_e(\omega) = \begin{cases} \frac{1}{P(e)} * \mu(\omega) & \text{if } \omega \models e \\ 0 & \text{if } \omega \not\models e \end{cases}$$

- **The conditional probability of formula h given evidence e is**

$$P(h \mid e) = \sum_{\omega \models h} \mu_e(w) = \frac{P(h \wedge e)}{P(e)}$$

Properties of Conditional Probabilities

- **Chain rule**

$$P(f_1 \wedge f_2 \wedge ... \wedge f_n)$$
$$= P(f_1) * P(f_2 \mid f_1) * P(f_3 \mid f_1 \wedge f_2) * ... * P(f_n \mid f_1 \wedge ... \wedge f_{n-1})$$
$$= \prod_{i=1}^{n} P(f_i \mid f_1 \wedge ... \wedge f_{i-1})$$

Bayes' theorem

- **The chain rule and commutativity of conjunction (h ∧ e is equivalent to e ∧ h) gives us:**

$$P(h \wedge e) = P(h|e) * P(e)$$
$$= P(e|h) * P(h).$$

- **If P(e) ≠ 0, you can divide the right hand sides by P(e).**

$$P(h \mid e) = \frac{P(e \mid h) * P(h)}{P(e)}$$

- **This is Bayes' theorem.**

Why is Bayes' theorem interesting?

- **Often you have causal knowledge:**

  P(symptom | disease)

  P(light is off | status of switches and switch positions)

  P(alarm | fire)

  P(image looks like  ***| a tree is in front of a car)***

- **and want to do evidential reasoning:**

  P(disease | symptom)

  P(status of switches | light is off and switch positions)

  P(fire | alarm)

  P(a tree is in front of a car | image looks like  ***)***